Measuring the Response of Macroeconomic Uncertainty to Shocks*

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Abstract

Recent research documents the importance of uncertainty in determining macroeconomic outcomes, but little is known about the transmission of uncertainty across such outcomes. This paper examines the response of uncertainty about inflation and output growth to shocks documenting statistically significant asymmetries and spillovers. Uncertainty about inflation is a determinant of output uncertainty, while higher growth volatility tends to raise inflation volatility. Both inflation and growth volatility respond asymmetrically to positive and negative shocks. Negative growth shocks and positive shocks to inflation lead to higher and more persistent uncertainty than shocks of equal magnitude but opposite sign.

Keywords: Asymmetry, Inflation, Output Growth, Variance Impulse Response Functions,

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1. Introduction

The role of uncertainty is central in many macroeconomic models explaining the dynamics of economic activity and inflation. Milton Friedman (1977), for example, has argued that uncertainty adversely affects the ability of the price mechanism to efficiently allocate resources; in Friedman's analysis, uncertainty regarding the realization of inflation is a contributing factor in slowing the rate of economic growth.[1] More recently, Huizinga (1993) argues that a greater degree of inflation uncertainty implies that actual realizations of inflation have a larger unexpected component, and could therefore have larger real effects. Hayford (2000) argues that high inflation will produce high inflation uncertainty in a world in which there is confusion regarding the monetary authority's predisposition towards lowering inflation. He identifies a spillover between uncertainty about inflation and real economic activity and shows that this can affect real output growth. Uncertainty also features in some models of the monetary policy transmission mechanism. Cukierman and Meltzer (1986) show that a monetary authority wishing to enact an expansionary policy can exploit inflation uncertainty; in effect the authority can use the uncertainty to cloak a high inflation policy in an attempt to boost economic activity. Links between growth uncertainty and real activity have also been hypothesized. Black (1987) suggests there will be a positive relation based on growth uncertainty being a time in which the riskiest investment projects become more profitable. Woodford (1990), however, hypothesizes a negative relation based on the increased riskiness of investment when output is volatile.[2]

In the empirical literature, researchers have generally adopted one of three approaches when modeling macroeconomic uncertainty. The first approach uses the unconditional second moments of the data. Examples include Logue and Willet (1976), Taylor (1981) and Ramey and Ramey (1995) *inter alia*. Other papers use the dispersion of survey forecasts of inflation and real activity to

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[1] See also Okun (1971).

[2] The empirical literature relating macroeconomic performance to growth and inflation uncertainty has produced mixed results. Numerous papers have tested for a link between output uncertainty and growth (Ramey and Ramey 1995, Kormendi and Meguire 1985, Grier and Tullock (1989, Caparale and McKiernan 1998, Grier and Perry 2001 and Henry and Olekalns 2002 *inter alia*). The results span the complete range from a negative to a zero to a positive relation. There is an equally large literature that relates inflation uncertainty and real output (see Holland 1993 for a survey). Here, there is a predominance of papers that find a negative relation (see Grier et. al. 2002 for a recent study).

proxy uncertainty (Cuikirman and Wachtel 1979, Hayford 2000, *inter alia*). Finally, there has been increasing use made of time series models of conditional heteroscedasticity (Engle 1982 and 1983, Jansen 1989, Henry and Olekalns 2002, *inter alia*).

In this paper, we provide a new empirical characterization of macroeconomic uncertainty by jointly modeling the conditional variance-covariance process underlying real economic activity and inflation.[3] Our approach improves on much of the previous research in allowing for the possibility of an asymmetric response of uncertainty to macroeconomic shocks and uncertainty spillovers across macroeconomic outcomes. Moreover, we make use of a new analytical tool, the Variance Impulse Response Function (VIRF); in our analysis, VIRFs (i) allow quantification of the extent to which uncertainties about real activity and inflation are interrelated, (ii) characterize the magnitude and persistence of macroeconomic uncertainty following a shock, and (iii) provide evidence of an asymmetric response of macroeconomic uncertainty to shocks. Unlike constant correlation models, commonly used in multivariate analysis, our approach has the advantage of not requiring the conditional correlation coefficient between real activity and inflation to be time invariant[4].

Our paper proceeds as follows. Section two describes the data, while the third section outlines the statistical model. Estimates and diagnostic tests are presented in the fourth section. The VIRFs are described in the penultimate section. The final section presents some concluding comments.

2. Data Description

The data used in this study were obtained from the FRED database at the Federal Reserve Bank of Saint Louis. The sample is monthly data over the period April 1947 to October 2000. We measure inflation, π_t, as the annualised, monthly difference of the logarithm of *P,* the producer price index;

[3] Our focus is on the conditional second moments of the data. This is appropriate for our purpose since one of our primary aims is to document the persistence of shocks to the variance-covariance process of inflation and real activity. This would not be possible if we worked with the unconditional second moments. Nor is there likely to be sufficient variation over time in survey-based measures of uncertainty to enable identification of the effects of shocks to the variance-covariance process.

[4] An example of a constant correlation approach to the joint modeling of real activity and inflation is the paper by Grier and Perry (2000)

$$\pi_t = \log\left(P_t / P_{t-1} \right) \times 1200 . \qquad (1)$$

Similarly we measure real activity as the annualised, monthly difference of the logarithm of *I*, the index of industrial production;

$$y_t = \log\left(I_t / I_{t-1} \right) \times 1200 . \qquad (2)$$

Table 1 presents summary statistics for the data. Both real activity and inflation are positively skewed and display significant amounts of excess kurtosis, with both series failing to satisfy the null hypothesis of the Bera-Jarque (1980) test for normality. Augmented Dickey-Fuller (1979) unit root tests and Kwiatkowski, Phillips, Schmidt and Shin (1992) tests for stationarity suggest that y_t and π_t are I(0) series. However, a series of Ljung-Box tests for serial correlation suggests that there is a significant amount of serial dependence in the data. Similarly, a Ljung-Box test for serial correlation in the squared data provides strong evidence of conditional heteroscedasticity in the data.

-Table 1 about here-

Since one of our concerns in this paper is to estimate an extremely general specification of the variance-covariance structure, we also present Engle and Ng's (1993) test for asymmetry in volatility in Table 1. This facilitates a test of *sign bias*; whether positive and negative shocks to volatility affect future volatility differently. *Size* bias, where not only the sign but also the magnitude of the innovation in volatility is important, can also be tested.

The results in Table 1 suggest that the conditional volatility of real activity may be sensitive to the size and sign of the innovation. There is strong evidence of negative size bias, some evidence of positive size bias, and the joint test for asymmetry in variance is highly significant at all usual levels of confidence. Likewise, the tests suggest that the sign of innovations to inflation influences inflation volatility with π_t displaying positive size bias. The joint test is significant at all usual levels of confidence.

Given the strong evidence in Table 1 of conditional heteroscedasticity and asymmetry in the data, we characterise the joint data generating process underlying inflation and real activity as a Multivariate Asymmetric GARCH-in-Mean model. The conditional mean equations of the model are modelled as an augmented Vector Autoregressive Moving Average or *VARMA(p,q)*,

$$Y_t = \mu + \sum_{i=1}^{p} \Gamma_i Y_{t-i} + \Psi \sqrt{h_t} + \sum_{j=1}^{q} \Theta_j \varepsilon_{t-j} + \varepsilon_t \qquad (4)$$

where $Y_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}; \varepsilon_t = \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \end{bmatrix}; \sqrt{h_t} = \begin{bmatrix} \sqrt{h_{y,t}} \\ \sqrt{h_{\pi,t}} \end{bmatrix}; \mu = \begin{bmatrix} \mu_y \\ \mu_\pi \end{bmatrix};\ \Gamma_i = \begin{bmatrix} \Gamma_{11}^{(i)} & \Gamma_{12}^{(i)} \\ \Gamma_{21}^{(i)} & \Gamma_{22}^{(i)} \end{bmatrix}; \Psi = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}$

and $\Theta_i = \begin{bmatrix} \theta_{11}^{(i)} & \theta_{12}^{(i)} \\ \theta_{21}^{(i)} & \theta_{22}^{(i)} \end{bmatrix}$.[5]

Under the assumption $\varepsilon_t \mid \Omega_t \sim (0, H_t)$, the model may be estimated using Maximum Likelihood methods, subject to the requirement that H_t be positive definite for all values of ε_t in the sample. This assumption of a symmetric time-varying variance-covariance matrix must be considered tenuous given the evidence in Table 1 documenting the asymmetric response of output and inflation volatility to positive and negative innovations of equal magnitude.

To allow for the possibility of asymmetric responses we extend the BEKK approach of Engle and Kroner (1995), using

$$H_t = C_0^{*'} C_0^* + A_{11}^{*'} \varepsilon_{t-1} \varepsilon_{t-1}^{'} A_{11}^* + B_{11}^{*'} H_{t-1} B_{11}^* + D_{11}^{*'} \xi_{t-1} \xi_{t-1}^{'} D_{11}^* \qquad (5)$$

where $C_0^* = \begin{bmatrix} c_{11}^* & c_{12}^* \\ 0 & c_{22}^* \end{bmatrix}; A_{11}^* = \begin{bmatrix} \alpha_{11}^* & \alpha_{12}^* \\ \alpha_{21}^* & \alpha_{22}^* \end{bmatrix}; B_{11}^* = \begin{bmatrix} \beta_{11}^* & \beta_{12}^* \\ \beta_{21}^* & \beta_{22}^* \end{bmatrix}; D_{11}^* = \begin{bmatrix} \delta_{11} & \delta_{21} \\ \delta_{12} & \delta_{22} \end{bmatrix}$ and

$$\xi_t^2 = \begin{bmatrix} \xi_{y,t}^2 \\ \xi_{\pi,t}^2 \end{bmatrix} = \begin{bmatrix} \min\{\varepsilon_{y,t}, 0\} \\ \max\{\varepsilon_{\pi,t}, 0\} \end{bmatrix}.$$

[5] We choose the values of p and q that minimise the Akaike and Schwarz information criteria. In the results below, $p=q=2$.

Note that $\xi_{y,t}$ allows for the observed negative sign and size bias in real activity and $\xi_{\pi,t}$ allows for the positive size bias in inflation. The inclusion of these variables, which can be interpreted as measuring the arrival of "bad news" regarding real activity and inflation, relaxes the assumption of a symmetric time-varying variance-covariance matrix.

4. Results and Specification Tests

Table 2 reports parameter estimates, as well as specification and diagnostic tests, for the full model given by (4) and (5). We follow Weiss (1986) and Bollerslev and Wooldridge (1992) who argue that asymptotically valid inference regarding normal quasi-maximum likelihood estimates may be based upon robustified versions of the standard test statistics.[6]

- Table 2 about here -

The inflation – real activity process is strongly conditionally heteroscedastic. The statistical significance of the off-diagonal elements of A^*_{11}, B^*_{11} and D^*_{11} matrices implies that innovations to inflation (real activity) significantly influence the conditional variance of real activity (inflation). The significance of the various elements of the D^*_{11} matrix implies that the sign as well as the size of both inflation and activity innovations are important.

The model appears to be well specified. The standardised residuals, $z_{it} = \varepsilon_{it}/\sqrt{h_{it}}$ for $i = y, \pi$, and their corresponding squares, satisfy the null of no fourth order linear dependence of the *Q(4)* and *Q^2(4)* tests. Similarly there is no evidence, at the 5% level, of twelfth order serial dependence in $z_{y,t}$ and $z^2_{y,t}$. On the basis of *Q^2(12)* though, there is some evidence of twelfth order dependence in the squared standardised residuals of inflation. For a well-specified model $E(z_{it}) = 0$ and $E(z^2_{it}) = 1$. These conditions are supported at any level of significance. The model also significantly reduces the degree of skewness and kurtosis in the standardised residuals when compared with the raw data.

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[6] The model was also estimated assuming a conditional Students-t distribution. The results were qualitatively unchanged. Details are available from the second author upon request.

Similarly the model predicts that $E\left(\varepsilon_{i,t}^2\right)=h_{i,t}$ for $i = y,\pi$ and $E\left(\varepsilon_{y,t}\varepsilon_{\pi,t}\right)=h_{y\pi,t}$. These conditions are supported by the data at the 5% level.

Table 3 reports the results of applying robust conditional moment bias tests to the estimated model (Kroner and Ng 1998). These tests are based on a comparison of the cross-product matrix of the residuals from the estimated model with the estimated covariance matrix. One indication that the estimated model provides a good characterization of the data is the absence of systematic patterns in the vertical distance between the elements of $\varepsilon_{y,t}\varepsilon_{\pi,t}$ and $h_{y\pi,t}$. This distance is measured by the generalized residual $v_{y\pi,t}=\varepsilon_{y,t}\varepsilon_{\pi,t}-h_{y\pi,t}$. A correctly specified model would imply $E_{t-1}(v_{y\pi,t})=0$; this means that $v_{y\pi,t}$ should be orthogonal to any variable known in period *t-1*.

We check for three types of systematic biases in the generalized residuals. For *sign* bias, we define indicator variables $m_1^i=I(\varepsilon_{i,t-1}<0)$ for $i=y,\pi$, where $I(\bullet)=1$ if the argument is true. A test for *quadrant* bias can be based on a partition of $\varepsilon_{yt-1}\varepsilon_{\pi t-1}$ according to $\left(\varepsilon_{y,t-1}<0,\ \varepsilon_{\pi,t-1}<0\right)$, $\left(\varepsilon_{y,t-1}>0,\ \varepsilon_{\pi,t-1}<0\right)$, $\left(\varepsilon_{y,t-1}<0,\ \varepsilon_{\pi,t-1}>0\right)$ and $\left(\varepsilon_{y,t-1}>0,\ \varepsilon_{\pi,t-1}>0\right)$. The indicator variables m_2 relate to these respective quadrants. Finally a set of indicators, m_3, can be defined that scale the sign bias indicators by the magnitude of the innovations. These variables can be used to detect sensitivity to the *sign and size* of the innovations.

-Table 3 about here-

Table 3 shows that, in the main, the model is well specified. Only two of the thirty generalised residual test statistics are significant at the 5% level. The indicator $m_3^{\pi,y}$, used to detect bias to the magnitude of $\varepsilon_{y,t-1}$ when $\varepsilon_{\pi,t}<0$ is significant for $v_{y,t}$. Similarly for the conditional variance of inflation only the indicator m_1^y is significant indicating some bias to forecasts of inflation volatility when growth innovations are negative. The conditional covariance equations display no evidence of quadrant and size/sign misspecification.

Finally, we note that all elements of the Ψ matrix are statistically significant at the 5% level. This is consistent with uncertainty about inflation and real activity impacting on the respective conditional means. The implications of this are discussed further in a companion paper to the current research (Grier et. al. 2002).

4. Variance Impulse Response Functions

In this section, we investigate the dynamics implied by the conditional variance-covariance structure of the model by perturbing the system with shocks to real activity and inflation. Specifically, we trace the effects of these shocks to the conditional variances (and covariance) through time, allowing for an asymmetric response (implied by expression (5)) to the shocks. The analysis extends the Generalised Impulse Response Functions (*GIR*'s) introduced by Koop *et al* (1996), in the context of multivariate non-linear systems, to the conditional variances of a system (as opposed to the traditional analysis of conditional means of series).

Since this technique is new, we now provide more detail. Define the random vector $Z_t = vech(H_t)$, where H_t is, as defined in section 3, the 2×2 conditional variance-covariance matrix of ε_t; Z_t will therefore be 3×1 dimensional vector, where the first, second and third elements are respectively given by $h_{y,t}$, $h_{y\pi,t}$ and $h_{\pi,t}$. The *VIRF* for a specific shock υ_t and history ω_{t-1} can then be given as,

$$VIRF_Z(n,\upsilon_t,\omega_{t-1}) = E[Z_{t+n} \mid \upsilon_t,\omega_{t-1}] - E[Z_{t+n} \mid \omega_{t-1}], \qquad (6)$$

for n = 0, 1, 2, …. Hence, the *VIRF* is conditional on υ_t and ω_{t-1} and constructs the response by averaging out future shocks given the past and present. Given this, a natural reference point for the impulse response function is the conditional expectation of Z_{t+n} given only the history ω_{t-1}, and, in this benchmark response, the current shock is also averaged out. Assuming that υ_t and ω_{t-1} are realisations of the random variables V_t and Ω_{t-1} that generate realisations of $\{Z_t\}$, then (following the ideas proposed in Koop et al (1996)) the *VIRF* defined in (6) can be considered to be a realisation of a random variable given by,

$$VIRF_Z(n, V_t, \Omega_{t-1}) = E[Z_{t+n} \mid V_t, \Omega_{t-1}] - E[Z_{t+n} \mid \Omega_{t-1}]\,. \qquad (7)$$

Note that the first and third elements of $VIRF_Z(n, V_t, \Omega_{t-1})$ give the impulse responses of the conditional variances of y_t and π_t, respectively, whilst the second element represents the impulse response relating to the conditional covariance.[7]

Analogous to *GIRF*s, a number of alternative conditional versions of the *VIRF*'s can be defined.[8] Given the asymmetric nature of the conditional variance-covariance structure, of particular interest is the evaluation of the significance of the asymmetric effects of positive and negative activity and inflation shocks on $h_{y,t}$, $h_{y\pi,t}$ and $h_{\pi,t}$. For instance, the response functions can be used to measure the extent to which negative shocks may (or may not) be more persistent than positive shocks as well as assess the potential diversity in the dynamics in the effects of positive and negative shocks on the conditional volatilities of output growth and inflation, and on their conditional covariance.

Let $VIRF_Z(n, V_t^+, \Omega_{t-1})$ denote the *VIRF* from conditioning on the set of all possible positive shocks, where $V_t^+ = \{\upsilon_t \mid \upsilon_t > 0\}$ and $VIRF_Z(n, -V_t^+, \Omega_{t-1})$ denote the *VIRF* from conditioning on the set of all possible negative shocks. The distribution of the random asymmetry measure,

$$ASY_Z(n, V_t^+, \Omega_{t-1}) = VIRF_Z(n, V_t^+, \Omega_{t-1}) - VIRF_Z(n, -V_t^+, \Omega_{t-1})\,, \qquad (8)$$

will be zero if positive and negative shocks have exactly the same effect. The distribution of (8) can provide an indication of the asymmetric effects of positive and negative shocks.

The asymmetry measure we propose is analogous to the measure proposed in van Dijk *et al* (2000) for the case of *GIRF*s. However, a notable distinction is that the measure in (8) is comprised of the *difference* between the variance response functions,

[7] Hafner and Herwartz (2001) also consider such an extension and derive analytical expressions for the *VIRF*'s for the case of symmetric multivariate GARCH models.

$VIRF_Z(n, V_t^+, \Omega_{t-1})$ and $VIRF_Z(n, -V_t^+, \Omega_{t-1})$, in contrast to the summation of the corresponding generalised impulse response versions. This distinction arises because *VIR*'s are made up of the *squares* of the innovations (and therefore will be of the same sign), in contrast to the case of *GIR*'s, where positive and negative shocks cause the response functions to take opposite signs. Note that the conditional variance-covariance structure proposed in this paper allows for asymmetry to enter through the terms $\xi_{y,t} = \min\{\varepsilon_{y,t}, 0\}$ and $\xi_{\pi,t} = \max\{\varepsilon_{2,t}, 0\}$, in the form of $\xi_{t-1}\xi_{t-1}^{'}$ in expression (5), where $\xi_t = \left(\xi_{y,t}\ \ \xi_{\pi,t}\right)^{'}$. Hence, if the matrix of coefficients, D_{11}^*, defined in (5) is not significantly different from zero, then the *VIRF* will not distinguish between a positive or negative shock. If, on the other hand, D_{11}^*, *is* significant, then the possibility of asymmetric responses to positive and negative shocks arises (even though $VIRF_Z(n, V_t^+, \Omega_{t-1})$ and $VIRF_Z(n, -V_t^+, \Omega_{t-1})$, relating to positive and negative shocks, respectively, will be of the *same* sign).

A second distinction between the *VIRF*s and *GIRF*s following naturally from this discussion is that, unlike for *GIRF'*s, the property of linearity in the shocks no longer holds. Therefore, a shock of $\kappa * \upsilon_t$, where κ is a scalar, will not have κ times the effect of υ_t, if we consider conditional volatility responses.

Finally, akin to *GIRF*s, *VIRF*s allow for composition dependence in multivariate models[9] and avoid problems of dependence on the size and sign of the shock. However, in contrast to *GIRF'*s, *VIRF*'s exhibit dependence on the history through the conditional variance-covariance matrix at time zero when the shock occurs (i.e. through $Z_0 = vech(H_0)$). This is clear from expression (5), setting *t=1.*

It is impossible to construct analytical expressions for the conditional expectations for the non-linear structure proposed in this paper. Therefore, Monte Carlo methods of stochastic simulation

[8] For instance, it is possible to condition on a particular shock and treat the variables generating the history as random, or, condition on a particular history and allow the shocks to be the random variables. Alternatively, particular subsets of shocks/histories could be conditioned on (see Koop *et al* for further details).

need to be used.[10] Following the algorithm described in Koop *et al*, impulse responses $VIRF_Z(n, \upsilon_t, \omega_{t-1})$ are computed for all 637 histories in the sample for horizons *n=0,1,…N*, with *N=50.* At each history, 100 draws are made from the joint distribution of the innovations and *R=250* replications are used to average out the effects of the shocks.[11]

- Figure 1 about here -

Figures 1, 2 and 3 display the VIRFs for real activity and inflation shocks bootstrapped from the data. Figure 1 displays the response functions for the conditional variances of activity and inflation to a shock that causes our activity measure to rise by a unit on impact. The activity shock results in a markedly higher and more persistent response from $h_{y,t}$ relative to $h_{\pi,t}$. The peak response of $h_{y,t}$ is approximately five times the maximum response of $h_{\pi,t}$ to the shock. Further, for activity volatility, the effects of the shock die out after roughly 40 months, while the effect of the growth shock on inflation volatility dissipates after approximately 15 months.

- Figure 2 about here -

The response of $h_{y,t}$ and $h_{\pi,t}$ to a shock that causes inflation to rise by one unit on impact is displayed in Figure 2. Here the peak response of $h_{\pi,t}$ is roughly twice the maximum response of $h_{y,t}$. Both responses appear to dissipate after approximately 20-25 months.

- Figure 3 about here -

The response functions for the conditional covariance to unit activity and inflation shocks are displayed in figure 3. The effect of the inflation shock causes real activity to move in opposite directions. Initially the covariance response is negative and significant although the effects die out within a year. On the other hand, the response to the activity shock is more volatile and dissipates within a year.

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[9] Hence, the effect of a shock to the conditional volatility of output growth, for example, is not isolated from having a contemporaneous impact on the conditional variance of inflation and vice versa. See Lee and Pesaran (1993) and Pesaran and Shin (1998) who consider composition dependence in (multivariate) conditional mean equations.

[10] See Granger and Teräsvirta (1993, Ch. 8), and Koop et al (1996) for detailed descriptions of the various methods that can be used.

Computation of the asymmetry measures for an activity (inflation) shock to the conditional variance and covariance of the activity and inflation series highlight the pernicious effects of bad news. A negative growth shock results in more persistent growth volatility (statistic=-0.13669, t-ratio = -27.5535), more persistence in covariance (statistic =-0.02083, t-ratio = -27.9532) and more persistence in inflation volatility (-0.026908, t-ratio=6.78044) than an unexpected positive activity shock of equal magnitude. Contractionary activity shocks lead to higher and more persistent uncertainty about inflation and activity.

Bad news about inflation, that is a shock that results in an increase to the inflation rate, leads to less activity volatility (statistic = -0.02664, t-ratio = -4.4769) more persistence in covariance (statistic = 0.188342, t-ratio 10.62642) and more persistence in inflation volatility (statistic = 1.2260, t-ratio = 10.97492) relative to an unanticipated reduction in inflation of equal magnitude.

In contrast to bad news about growth, an unexpected inflationary shock actually leads to less, rather than more, persistence in growth volatility. This may be suggestive of the stabilising effects of monetary policy in response to an increase in inflation and inflation uncertainty. However, in general these asymmetry measures are small in magnitude relative to the size of the initial shock and are therefore perhaps unlikely to be of great economic significance. The possible exception is the measure of asymmetry in the response of inflation volatility to an inflationary shock. This measure is sufficiently large to have economic and statistical significance. Bad news about inflation leads to a higher level of inflation and more inflation volatility.

6. Conclusions

In this paper, we provide an extremely general empirical characterization for real economic activity and inflation. Particular attention has been paid to estimating a fairly unrestricted specification for the conditional second moments of the data. Given the central role that uncertainty plays in many macroeconomic models, it is of primary importance that second moment restrictions not supported

[11] Note that the number of replications (R) is set to a relatively small number given the hugely time-consuming nature of the computations. However, the large number of histories employed, and the precision with which the impulse response

by the data be avoided at all costs in macroeconometric modeling. Indeed, we find that GARCH models of inflation and real activity will be misspecified unless asymmetries and uncertainty spillovers are incorporated into the empirical specification. Failure to do so must raise concerns about any inferences made on the basis of these models.

An additional contribution of the paper has been to document considerable persistence in the response of uncertainty to macroeconomic shocks using Variance Impulse Response Functions. For example, it can take up to three years before the uncertainty generated by a shock to economic activity dissipates. To our knowledge, this persistence in uncertainty has not been documented elsewhere.

Finally, our Variance Impulse Response analysis demonstrates that macroeconomic uncertainty responds asymmetrically to macroeconomic shocks, with the arrival of bad news having a more significant effect on uncertainty than good news. Bad news, that is positive shocks to inflation and negative shocks to growth, leads to higher and more persistent volatility than would result from good news of similar magnitude. In particular, inflation uncertainty displays an asymmetric response to inflationary shocks that is both statistically significant and economically meaningful.

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functions have been estimated, both serve to mitigate the concerns over the size of *R*..

Table 1: Summary Statistics

	Mean	Variance	Skewness	Excess Kurtosis	Bera-Jarque Normality
y	3.6054	155.7047	0.2428	4.5962	562.4889 [0.0000]
π	3.0559	37.5103	1.1579	4.4310	658.2563 [0.0000]
			Unit Root and Stationarity Tests		
	ADF(μ)	**ADF(τ)**	**ADF**	**KPSS(μ)**	**KPSS(τ)**
y	-12.4483	-12.4438	-11.6179	0.07595	0.03498
π	-5.4309	-5.3842	-4.3728	0.4664	0.3975
5 % C.V.	-3.4191	-2.8664	-1.9399	0.463	0.146
			Tests for Serial Correlation and ARCH		
	Q(4)	***Q(12)***	***Q^2(4)***	***Q^2(12)***	**ARCH(4)**
y	165.3173 [0.0000]	192.0829 [0.0000]	88.1327 [0.0000]	97.4497 [0.0000]	52.1685 [0.0000]
π	321.3849 [0.0000]	682.6248 [0.0000]	136.8077 [0.0000]	463.0983 [0.0000]	62.7177 [0.0000]
			Tests for Asymmetry in Variance		
	Sign	**Neg. Size**	**Pos. Size**	**Joint**	
y	0.2418 [0.0159]	-7.19740 [0.0000]	3.2857 [0.0011]	83.3489 [0.0000]	
π	-0.9672 [0.3338]	0.5698 [0.5690]	8.6105 [0.0000]	72.2217 [0.0000]	

Note: Marginal significance levels displayed as [.]

Table 2: The Multivariate Asymmetric GARCH-in-Mean Model

Conditional Mean Equations

$$Y_t = \mu + \sum_{i=1}^{p} \Gamma_i Y_{t-i} + \Psi \sqrt{h_t} + \sum_{j=1}^{q} \Theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$Y_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} \; ; \; \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \; ; \; \Gamma_i = \begin{bmatrix} \Gamma_{11}^i & \Gamma_{12}^i \\ \Gamma_{21}^i & \Gamma_{22}^i \end{bmatrix} \; ; \; \Psi = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix};$$

$$\sqrt{h_t} = \begin{bmatrix} \sqrt{h_{y,t}} \\ \sqrt{h_{\pi,t}} \end{bmatrix}; \varepsilon_t = \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \end{bmatrix}; \Theta_j = \begin{bmatrix} \theta_{11}^j & \theta_{12}^j \\ \theta_{21}^j & \theta_{22}^j \end{bmatrix}$$

$$\mu = \begin{bmatrix} 1.2584 \\ (0.0545) \\ \\ 0.0913 \\ (0.0172) \end{bmatrix} \qquad \Gamma_1 = \begin{bmatrix} 0.4385 & 0.04768 \\ (0.0121) & (0.0102) \\ & \\ 0.0072 & 0.7794 \\ (0.0053) & (0.0047) \end{bmatrix} \qquad \Gamma_2 = \begin{bmatrix} 0.3339 & -0.1126 \\ (0.0117) & (0.0109) \\ & \\ 0.0233 & 0.1939 \\ (0.0045) & (0.0046) \end{bmatrix}$$

$$\Theta_1 = \begin{bmatrix} -0.2525 & -0.1897 \\ (0.0243) & (0.0467) \\ & \\ 0.0012 & -0.6225 \\ (0.0085) & (0.0246) \end{bmatrix} \qquad \Theta_2 = \begin{bmatrix} -0.3131 & 0.0170 \\ (0.0274) & (0.0559) \\ & \\ -0.0171 & -0.2006 \\ (0.0075) & (0.0241) \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0.0846 & -0.2385 \\ (0.0065) & (0.0113) \\ & \\ -0.0036 & -0.0209 \\ (0.0017) & (0.0037) \end{bmatrix}$$

Residual Diagnostics

	Mean	Variance	*Q(4)*	$Q^2(4)$	*Q(12)*	$Q^2(12)$
$\varepsilon_{1,t}$	0.0140	0.9932	2.8898	6.1466	21.4150	11.7959
	[0.7225]	[0.9969]	[0.5764]	[0.1885]	[0.0446]	[0.4622]
$\varepsilon_{2,t}$	0.0265	1.0088	1.9639	5.6143	11.4304	26.9583
	[0.5035]	[0.9991]	[0.7474]	[0.2298]	[0.4924]	[0.0078]

Moment Based Tests

$E(\varepsilon_{y,t}^2) = h_{y,t}$	$E(\varepsilon_{\pi,t}^2) = h_{\pi,t}$	$E(\varepsilon_{y,t}\varepsilon_{\pi,t}) = h_{y\pi,t}$
0.6317	3.6123	2.0114
[0.4267]	[0.0574]	[0.1561]

Table 2 Continued: Estimates of the Multivariate Asymmetric GARCH Model

Conditional Variance-Covariance Structure

$$H_t = C_0^{*'} C_0^* + A_{11}^{*'} \varepsilon_{t-1} \varepsilon_{t-1}^{'} A_{11}^* + B_{11}^{*'} H_{t-1} B_{11}^* + D_{11}^{*'} \xi_{t-1} \xi_{t-1}^{'} D_{11}^*$$

$$\varepsilon_{t-1} = \begin{bmatrix} \varepsilon_{y,t-1} \\ \varepsilon_{\pi,t-1} \end{bmatrix} \; ; \; \xi_{t-1} = \begin{bmatrix} \min(\varepsilon_{y,t-1}, 0) \\ \max(\varepsilon_{\pi,t-1}, 0) \end{bmatrix}$$

$$C_0^* = \begin{bmatrix} \begin{matrix} 1.8064 \\ (0.0817) \end{matrix} & \begin{matrix} 0.6612 \\ (0.1595) \end{matrix} \\ 0 & \begin{matrix} 1.2033 \\ (0.0977) \end{matrix} \end{bmatrix} \qquad B_{11}^* = \begin{bmatrix} \begin{matrix} 0.9155 \\ (0.0026) \end{matrix} & \begin{matrix} 0.0024 \\ (0.0213) \end{matrix} \\ \begin{matrix} -0.1414 \\ (0.1088) \end{matrix} & \begin{matrix} -0.8567 \\ (0.0064) \end{matrix} \end{bmatrix}$$

$$A_{11}^* = \begin{bmatrix} \begin{matrix} -0.0741 \\ (0.0255) \end{matrix} & \begin{matrix} 0.0627 \\ (0.0139) \end{matrix} \\ \begin{matrix} 0.0202 \\ (0.0818) \end{matrix} & \begin{matrix} 0.3844 \\ (0.0179) \end{matrix} \end{bmatrix} \qquad D_{11}^* = \begin{bmatrix} \begin{matrix} -0.5711 \\ (0.0147) \end{matrix} & \begin{matrix} 0.0123 \\ (0.0176) \end{matrix} \\ \begin{matrix} 0.3409 \\ (0.0745) \end{matrix} & \begin{matrix} 0.2479 \\ (0.0518) \end{matrix} \end{bmatrix}$$

Diagonal VARMA	$H_0 : \Gamma_{12}^i = \Gamma_{21}^i = \theta_{12}^i = \theta_{21}^i = 0$	[0.0000]
No GARCH-M	$H_0 : \psi_{ij} = 0$ for all i, j	[0.0000]
No asymmetry:	H_0: $\delta_{ij}=0$ for $i,j=1,2$	[0.0000]
Diagonal GARCH	$H_0 : \alpha_{12}^* = \alpha_{21}^* = \beta_{12}^* = \beta_{21}^* = \delta_{12}^* = \delta_{21}^* = 0$	[0.0000]

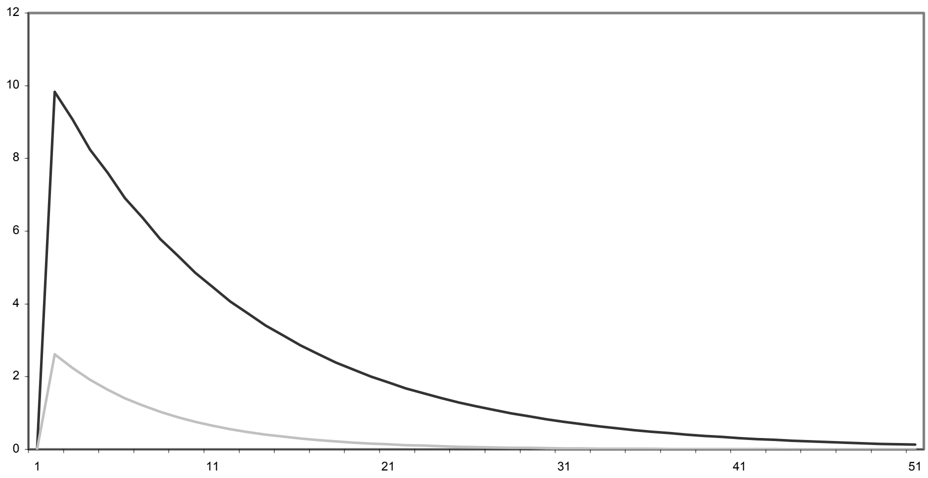
Notes: Standard errors displayed as (.). Marginal significance levels displayed as [.]. *Q(p)* and $Q^2(p)$ are Ljung_Box tests for p[th] order serial correlation in $z_{j,t}$ and $z_{j,t}^2$ respectively for $j = y_t, \pi_t$.

Table 3: Robust Conditional Moment Tests

Indicator	$v_{y,t}=\varepsilon_{y,t}^2-h_{y,t}$	$v_{y\pi,t}=\varepsilon_{y,t}\varepsilon_{\pi,t}-h_{y\pi,t}$	$v_{\pi,t}=\varepsilon_{\pi,t}^2-h_{\pi,t}$
m_1^y	0.2002 [0.6546]	1.1889 [0.2756]	6.0239 [0.0014]
m_1^π	0.0007 [0.9789]	0.5253 [0.4686]	0.1048 [0.7461]
$m_2^{-,-}$	4.4018 [0.0359]	0.4363 [0.5089]	0.2990 [0.5845]
$m_2^{-,+}$	0.8892 [0.3457]	2.4581 [0.1169]	1.2379 [0.2659]
$m_2^{+,-}$	1.2342 [0.2666]	1.4946 [0.2215]	1.4946 [0.2215]
$m_2^{+,+}$	0.0004 [0.9844]	0.1814 [0.6701]	1.7098 [0.1910]
$m_3^{y,y}$	0.1471 [0.7014]	1.5014 [0.2204]	4.3499 [0.0370]
$m_3^{y,\pi}$	0.1358 [0.7125]	0.1792 [0.6721]	3.2139 [0.0730]
$m_3^{\pi,y}$	0.8974 [0.3435]	0.0001 [0.9941]	0.5373 [0.4636]
$m_3^{\pi,\pi}$	0.7223 [0.3954]	0.6679 [0.4138]	1.0869 [0.2972]

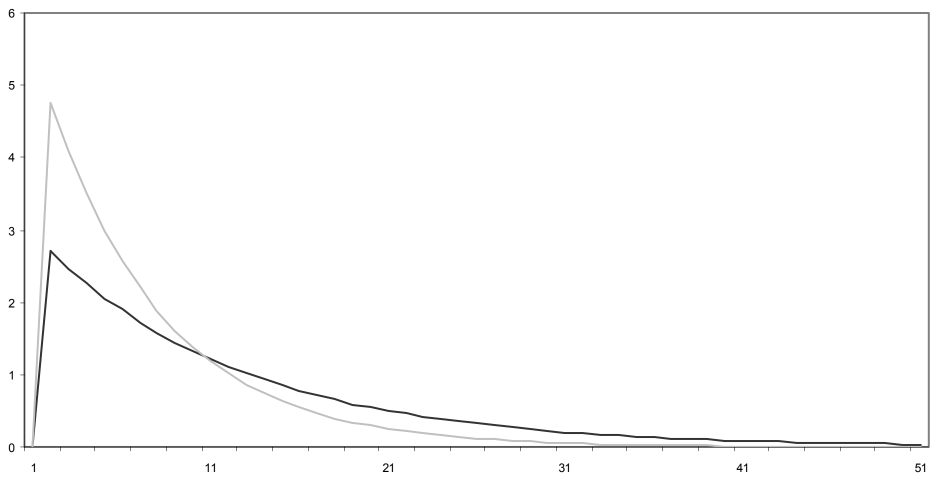
Sign Misspecification	Quadrant Misspecification	Size/ Sign Misspecification
$m_1^y=I\left(\varepsilon_{y,t-1}<0\right)$	$m_2^{-,-}=I\left(\varepsilon_{y,t-1}<0,\varepsilon_{\pi,t-1}<0\right)$	$m_3^{y,y}=\varepsilon_{y,t-1}^2 I\left(\varepsilon_{y,t-1}<0\right)$
$m_1^\pi=I\left(\varepsilon_{\pi,t-1}<0\right)$	$m_2^{+,-}=I\left(\varepsilon_{y,t-1}>0,\varepsilon_{\pi,t-1}<0\right)$	$m_3^{y,\pi}=\varepsilon_{y,t-1}^2 I\left(\varepsilon_{\pi,t-1}<0\right)$
	$m_2^{-,+}=I\left(\varepsilon_{y,t-1}<0,\varepsilon_{\pi,t-1}>0\right)$	$m_3^{\pi,y}=\varepsilon_{\pi,t-1}^2 I\left(\varepsilon_{y,t-1}<0\right)$
	$m_2^{+,+}=I\left(\varepsilon_{y,t-1}>0,\varepsilon_{\pi,t-1}>0\right)$	$m_3^{\pi,\pi}=\varepsilon_{\pi,t-1}^2 I\left(\varepsilon_{\pi 2,t-1}<0\right)$

Notes: All tests are distributed as $\chi^2(1)$. Marginal significance levels displayed as [.]. The misspecification indicator is defined where *I(*)* takes the value 1 if the expression in the parentheses below is satisfied and zero otherwise.



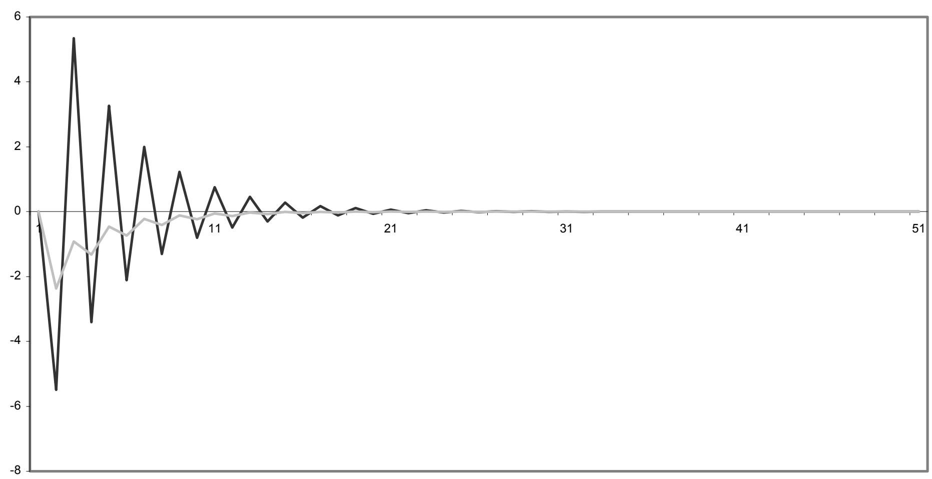
$h_{y,t}$ $h_{\pi,t}$

Figure 1: VIRF for a unit growth shock on $h_{y,t}$ and $h_{\pi,t}$



$h_{y,t}$ $h_{\pi,t}$

Figure 2: VIRF for a unit inflation shock on $h_{y,t}$ and $h_{\pi,t}$



y shock to $h_{y\pi,t}$ π shock to $h_{y\pi,t}$

Figure 3: VIRF for a unit growth and inflation shock on $h_{y\pi,t}$

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