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Choosing Longevity with Overlapping Generations

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Abstract

We extend Diamond's (1965) OLG model to allow agents to choose whether to participate in the second period of life. The valuation of early exit (x) is a key parameter. We characterize competitive equilibria, efficient allocations, and predictions for income and life expectancy over time. We find that, with logarithmic utility, for any value of x , there is a range of initial values of the capital stock for which some agents would prefer to exit in equilibrium. The shape of the transition function and the number of steady state equilibria depend crucially on the value of capital's share of income.

Key words: Endogenous longevity, overlapping generations, growth.

JEL Numbers: D91, O1

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“There is but one truly serious philosophical problem, and that is suicide.
Judging whether life is or is not worth living amounts to answering
the fundamental question of philosophy.”

Albert Camus

“To be, or not to be: that is the question:
Whether 'tis nobler in the mind to suffer
The slings and arrows of outrageous fortune,
Or to take arms against a sea of troubles,
And by opposing end them? To die: to sleep;
No more; and by a sleep to say we end
The heart-ache and the thousand natural shocks
That flesh is heir to, 'tis a consummation
Devoutly to be wish'd. To die, to sleep;
To sleep: perchance to dream: ay, there's the rub;
For in that sleep of death what dreams may come
When we have shuffled off this mortal coil...”

William Shakespeare

“Live long and prosper!”

Spock

1. INTRODUCTION

At the most basic level of decision-making, each of us must decide whether or not to continue living (“to be, or not to be”). Moreover, decisions we make early in our lives can have a significant impact on how long we expect to live. Thus, either directly or indirectly, people do *choose* their expected longevity. Recent empirical work, by Smith *et al* (2001), supports the position that individuals are well aware of the implications of their decisions on longevity. This choice can potentially influence savings, investment, and growth. Although this may be quite well understood at an intuitive level, until recently, this choice, and its effects on growth and welfare, have not been analysed directly by economists.

In this paper we examine this choice directly in a very simple two-period overlapping generations model with production (based on Diamond (1965)) which is standard in every way except that individuals have the ability to choose whether or not to be alive

in the second period of their lives. A crucial element in the model is the agents' assessment of what happens to their utility if they choose not to live in the second period ("what dreams may come when we have shuffled off this mortal coil"). We denote this by a parameter: x . If the value of this parameter is large enough, relative to the utility they would achieve in their second period in equilibrium if they stay alive ("the slings and arrows of outrageous fortune") then (at least) some agents would prefer to exit ("to take arms against a sea of troubles, and by opposing end them"). With logarithmic preferences, as we consider here, there always exists a critical value of $x \in \mathbb{R}$, above which some would make this choice.

We show that the general equilibrium consequences of this choice, and its implications for growth, can be quite significant. In particular, in the standard model, if the initial value of capital per worker is small enough, then some agents would choose to exit – no matter what value x takes – along the transition path. Moreover, whenever some agents choose to exit, the number of steady state equilibria depends critically on the value of capital's share of income α . If $\alpha < 0.5$ then the equilibrium transition function is concave, and only one stable steady state equilibrium exists – with a strictly positive value of the capital stock. If $\alpha > 0.5$ then the equilibrium transition function is convex for low values of the capital stock, and the steady state equilibrium at the origin is stable. If x is high enough, then the equilibrium at the origin is the *unique* steady state equilibrium. Otherwise, a locally stable steady state equilibrium, with a positive value of the capital stock, also exists.

In general, regardless of the value of α , in the equilibria of this model, higher values of capital and income are associated with longer average life expectancy. This positive relationship between these two variables is something commonly observed in empirical studies¹ and reflects a two-way causality in the model: when individuals

¹ See, for example, the classic work by Preston (1975), the survey by Deaton (2003), and the recent study by Soares (2007).

earn more income, they choose to live longer, and when individuals choose to live longer they choose to save more – which increases income in general equilibrium (“live long and prosper”).

We also characterize efficient allocations, where a planner maximizes surplus in steady states by choosing consumption allocations, but where individuals have the choice of whether or not to live in the second period. Once again, the value of x is a crucial determinant of the optimal allocations. If x is below a threshold value then the planner would allocate consumption to ensure that all individuals live in the second period. Otherwise, the efficient allocation is one in which all individuals choose to exit. In particular, we show that it is never efficient for some individuals to exit while others do not, in the steady state. Finally, we show that the key results of the model are robust to two extensions: allowing agents to choose exit probabilities for the second period, and introducing public health in a way similar to Chakraborty’s (2004) study.

Related Literature

Most commonly, when considering the issue of longevity, theorists assume that an individual’s expected lifetime is independent of his decisions. Following Yaari (1965), for example, many researchers have modelled this as a parametric probability of surviving from one period to the next. This assumption allows for comparative dynamic analysis of the effects of changes in this parameter upon growth.² Other researchers have acknowledged the endogeneity of this probability and incorporated it into their theory by making it a function of aggregate variables, which are beyond the control of individuals. For example, Chakraborty (2004) models this probability as being a function of the level of public health expenditures.³

² See, for example, Ehrlich and Lui (1991), Kalemli-Ozcan (2002), and Zhang *et al* (2003).

³ See, also, Blackburn and Issa (2002).

Some research has concentrated on the choices that people make that affect their longevity, but without drawing out the implications for growth. For example, Grossman (1972) developed a continuous time model in which each individual invests in her own health capital, and where death occurs when health capital falls below a certain threshold. Ehrlich and Chuma (1990), in an econometric study, specified a demand function for longevity.

Blackburn and Cipriani (2002) analysed the decision process in a three-period overlapping generations model with human capital, fertility, and child-rearing. In that paper, the probability of survival is modelled as a behavioural function, with the level of human capital as its sole argument. Thus, expected longevity is determined indirectly through human capital choices. Becsi (2003) tackles the choice directly in a continuous time overlapping generations model, with Yaari-style survival probabilities, but where the terminal date of the planning horizon for the consumer is a choice variable. He finds that no internal solution can be found unless more structure is imposed on the model. In particular, to get an internal solution, he introduces the unusual assumption that longevity is a decreasing function of energy consumption. Bhattacharya and Qiao (2005) examine a model with both individual and public health expenditures, and a behavioural function transforming these expenditures into length of life. Finlay (2006) uses a model similar to that in Chakraborty (2004), but where health expenditures are chosen by individuals (rather than by a public health system) and where human capital is the engine of endogenous growth. In her model, life expectancy is chosen indirectly through health expenditures, which are then translated into survival probabilities using the behavioural function used in Blackburn and Cipriani (2002), but where health expenditure (rather than human capital) is the argument in the function. These last two environments are quite complex, and numerical methods are used to analyse the equilibrium outcomes.

In this paper we are able to derive analytical solutions for the equilibrium transition function, and easily characterize behaviour in the steady state, by using a simple framework with logarithmic preferences, a Cobb-Douglas production technology, and complete depreciation. In an appendix, we also analyse a more general case, with incomplete depreciation.

The remainder of the paper is structured as follows. Section 2 introduces the model and defines the equilibrium concept used here. Section 3 characterizes the properties of the equilibrium, and analyses its comparative dynamics. Section 4 analyses efficient allocations. Section 5 presents two extensions of the base model. Conclusions and suggestions for further research are presented in Section 6. Proofs of some of the propositions are presented in an Appendix A, and the extension using arbitrary depreciation values is contained in Appendix B.

2. THE MODEL

Time is discrete, agents live for (at most) two periods, and generations overlap. In each time period, a constant number (normalized to unity) of young agents is born. Each agent within any generation is identical *ex ante*. As is standard, we refer to agents born in period t as “young agents” and those surviving through period $t+1$ as “old agents”. All agents are endowed with one unit of labour when young, and none when old. Each young agent in period t chooses whether or not to exit life (terminate her life) at the end of period t . Apart from this decision to exit, the model is the same as Diamond’s (1965) growth model.

The Young Agents’ Problem

The novel feature of the analysis is the decision to exit life. Let $I_t \in \{0,1\}$ denote an indicator function, where $I_t = 1$ indicates a decision by agent t made in period t to exit

life at the end of period t (i.e. at the end of the period of youth), and $I_t = 0$ indicates the decision not to exit.⁴

We represent utility of agents born in period $t = 0,1,2,\dots$ as follows:

$$U_t = \begin{cases} u(c_{1t}) + \beta u(c_{2t+1}) & \text{if } I_t = 0 \\ u(c_{1t}) + x & \text{if } I_t = 1 \end{cases} \quad (1)$$

where c_{1t} is consumption when young, and c_{2t+1} is consumption when old provided that the agent does not exit. Utility is time separable, where $u(c)$ is the utility from consuming in the period and $\beta \in (0,1)$ is the discount factor.

Here the parameter $x \in \mathbb{R}$ is the agent's perception of the value, in utils, of exiting life early. Thus, x can be interpreted as the opportunity cost of living long. We introduce this parameter explicitly because we study circumstances where exiting may be the most palatable choice.⁵ We assume that all agents have a common perception of this value.

Throughout the paper we also restrict attention to logarithmic utility:

$$u(c) = \ln c.$$

We choose this specification not only because it is standard, and easy to work with, but also because the decision to exit is plausible when agents are poor and not infinitely averse to the prospect of an early death. With more general utility, it is

⁴ Modelling the exit choice as discrete is not restrictive in this environment. In Section 5 we consider the problem of allowing agents to choose a probability of exit, from $[0,1]$. We show that the choice of a corner is always superior to any interior solution.

⁵ Blackburn and Cipriani (2002), Becsi (2003), Chakraborty (2004) and Finlay (2006) all set $x = 0$, either implicitly or explicitly. This is a harmless normalization when individual actions do not directly affect the probability of exit, but not here – where this choice is the central focus of the study. Hence, we prefer to keep this general. This also allows for an examination of the consequences of changes in x .

straightforward to show that no agent would ever choose to exit if $\beta u(0) \geq x$. This is ruled out by the logarithmic specification.

The period constraints for youth and old age are, respectively, $c_{1t} + s_t = w_t$ and $c_{2t+1} = R_{t+1}s_t$, where $R_{t+1} = 1 + r_{t+1}$ is the gross interest rate and s_t is savings of the young. If an agent exits we assume for simplicity that any savings made at time t is discarded.⁶

Recall that exit occurs at the end of the period of youth. Agents that exit face no future decisions. Agents that do not exit trivially choose to consume $c_{2t+1} = R_{t+1}s_t$, in the usual way, in old age. Knowing this, agents born in period t choose in youth s_t , c_{1t} , and I_t , to maximize utility U_t subject to $c_{1t} + s_t = w_t$. The exit choice is discrete, so we consider the two possible cases.

Case $I_t = 0$

This is a standard life-cycle consumption problem, with the solutions:

$$s_t = \frac{\beta}{1+\beta} w_t \quad c_{1t} = \frac{1}{1+\beta} w_t \quad c_{2t+1} = \frac{\beta}{1+\beta} w_t R_{t+1} \quad (2)$$

Substitution of (2) into (1) yields the maximized value function:

$$V_{0t} = \ln\left(\frac{w_t}{1+\beta}\right) + \beta \ln\left(\frac{\beta w_t R_{t+1}}{1+\beta}\right) \quad (3)$$

⁶ Of course, in this discrete case, no agent ever chooses both exit and positive savings. We also get the same general results if we use an annuity market as in Chakraborty (2004). Under either specification, the interest rate is exogenous to individuals and thus individual choice is generically similar.

Case $I_t = 1$

In this case, to maximize (1), the agent sets $s_t = 0$, and so $c_{1t} = w_t$. This implies a maximized value function:

$$V_{1t} = \ln w_t + x \quad (4)$$

This leads to the following lemma.

Lemma 1: Given the wage rate $w_t > 0$ and the gross interest rate $R_{t+1} > 0$, young workers choose not to exit life, $I_t = 0$, at the end of the first period if and only if:

$$x \leq \beta \ln w_t + \beta \ln R_{t+1} + \beta \ln \beta - (1 + \beta) \ln(1 + \beta) \quad (5)$$

Proof: Agents will choose $I_t = 0$ if and only if $V_{0t} \geq V_{1t}$. Using equations (3) and (4) in this condition, one obtains (5). ■

Agents are indifferent between exiting or not when (5) holds with equality. Figure 1 identifies a locus of points in (w_t, R_{t+1}) where agents are indifferent about the two choices. It is easy to show that this locus is a rectangular hyperbola. Also, an increase in x (i.e. the value of exiting early) shifts the locus outwards indicating that individuals need higher wages or interest rates to choose not to exit.

The Firms' Problem

The firms' problem in this model is entirely standard. There are many competitive firms in this economy, and the number is normalized to unity. In any period t , each firm takes $\{w_t, r_t\}$ as given and solves the following problem:

$$\underset{\{Y_t, K_t, L_t\}}{\text{Max}} \Pi_t = Y_t - (r_t + \delta)K_t - w_t L_t$$

subject to the production technology:

$$Y_t = AK_t^\alpha L_t^{1-\alpha}$$

where $A > 0$, and $\alpha \in (0,1)$. To derive simple closed form solutions we assume 100% depreciation, $\delta = 1$.⁷ Defining capital per worker $k_t \equiv K_t / L_t$, the firm's problem leads to following standard first order-conditions in intensive form:

$$w_t = (1-\alpha)Ak_t^\alpha \tag{6}$$

$$R_{t+1} = \alpha Ak_{t+1}^{\alpha-1} \tag{7}$$

Equilibrium

Let p_t denote the fraction of young workers, in period t , who choose $I_t = 0$, i.e. not to exit. In any interior solution, where workers are indifferent about choosing to exit or not, p_t is determined where condition (5) holds with equality. The capital market equilibrium condition is influenced by p_t : since only those agents who do *not* exit save, p_t is also the proportion of savers. Savers provide the capital for period $t+1$. In equilibrium this supply must be equal to the demand for capital by firms:

$$k_{t+1} = p_t s_t \tag{8}$$

The labour market equilibrium condition in this model is perfectly standard:

⁷ The general case with $\delta \in [0,1]$ is developed in Appendix B. Depreciation does not affect our generic results for the number and stability of equilibria. However, it does affect the shape of the implied Preston curve, as discussed below.

$$L_t = 1 \tag{9}$$

Each young agent supplies one unit of labour inelastically, the number of agents in each generation is normalized to unity and, in equilibrium, this is equal to the demand for labour from firms.

A competitive equilibrium in this model, given k_0 , is a set of wages, interest rates, and fractions of savers $\{w_t, r_t, p_t\}$ and a set of allocations $\{c_{1t}, c_{2t}, I_t, s_t, k_t\}$ such that

- a) Individuals are maximizing utility (1) given the budget constraints, wages, and interest rates, with behaviour given in equations (2)-(5) and Lemma 1.
- b) Firms are choosing capital and labor to maximize profits, subject to the production technology, wages and interest rate (equations (6) and (7) are satisfied).
- c) Supply equals demand in the factor markets: equations (8) and (9) are satisfied.

3. PROPERTIES OF THE EQUILIBRIUM

For a given proportion of savers, p_t , we can derive the following equilibrium transition function from equations (2), (6), (8) and (9):

$$k_{t+1} = p_t \frac{\beta}{1+\beta} (1-\alpha) A k_t^\alpha \tag{10}$$

The following lemma establishes that the equilibrium proportion of savers is strictly less than unity for small values of k_t , and increasing until a threshold level \tilde{k} is achieved, at which point $p_t = 1$, and all agents choose not to exit the economy.

Lemma 2: For any given configuration of parameters (α, β, A, x) , the equilibrium relationship between the proportion of savers p_t (i.e. those that do not exit) and capital per worker k_t satisfies:

$$p_t = p_t^* \in (0,1) \Leftrightarrow 0 < k_t < \tilde{k} \quad (11a)$$

$$p_t = 1 \Leftrightarrow k_t \geq \tilde{k}, \quad (11b)$$

$$p_t = 0 \Leftrightarrow k_t = 0 \quad (11c)$$

where $p_t^* = \left(\frac{\rho_2}{\rho_1}\right) \left[(1-\alpha)Ak_t^\alpha\right]^{\frac{\alpha}{1-\alpha}}$ is the internal proportion of savers,

$\tilde{k} = \left[\frac{1}{(1-\alpha)A} \left(\frac{\rho_1}{\rho_2}\right)^{\frac{1-\alpha}{\alpha}} \right]^{\frac{1}{\alpha}} > 0$ is a threshold level of per worker capital,

and coefficients $\rho_1 \equiv \frac{\beta}{1+\beta} > 0$, $\rho_2 \equiv \frac{\beta^{\frac{1}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}}}{e^{\frac{x}{\beta(1-\alpha)}} (1+\beta)^{\frac{1+\beta}{\beta(1-\alpha)}}} > 0$.

Proof: See Appendix A.

Notice that this lemma tells us that the threshold value \tilde{k} is positive for all $x \in \mathbb{R}$. By (11a), this implies that $p_t < 1$ for some range $k_t \in (0, \tilde{k})$. It is also easy to show that $\partial \rho_2 / \partial x < 0$ and so $\partial \tilde{k} / \partial x > 0$. Intuitively, as the value of exiting life early increases, the critical value of k for which all individuals choose not to exit early also increases: the economy must offer more to individuals if they are to choose not to exit.

We are now in a position to characterize the equilibrium transition function. Substitution of the equilibrium values of p_t described in Lemma 2 into equation (10) yields the following:

$$k_{t+1} = \begin{cases} \rho_2 ((1-\alpha)Ak_t^\alpha)^{\frac{1}{1-\alpha}}, & \forall 0 \leq k_t < \tilde{k} & (12a) \\ \rho_1 (1-\alpha)Ak_t^\alpha, & \forall k_t \geq \tilde{k} & (12b) \end{cases}$$

Equation (12b) is the standard equilibrium transition function from Diamond's model, with logarithmic preferences and Cobb-Douglas production technologies. In our model it describes the equilibrium only for values of k greater than the threshold \tilde{k} , beyond which no-one chooses to exit: $p_t = 1$. Equation (12a) is the transition function when agents are indifferent to exiting or not – derived directly by substitution of p_t^* from equation (11a) into equation (10).

Figures 2 and 3 illustrate the equilibrium transition function as the inner envelope of the loci (a) and (b) which corresponds to equations (12a) and (12b) respectively. These two diagrams are drawn under different assumptions about the value of α , as described in the next paragraph. In both diagrams, however, the transition function is represented by the solid line. Locus (a) applies along the range $k_t \in (0, \tilde{k})$, and locus (b) applies for all $k_t \geq \tilde{k}$. Along the transition path, the proportion of savers $p_t = p_t^* < 1$ increases with k_t until $k_t = \tilde{k}$ is reached, after which, $p_t = 1$.⁸

The shape of the transition path depends on the value of α . Locus (b) is clearly concave, but the curvature of locus (a) depends on α . If $\alpha \leq 0.5$, then the locus (a) is concave, and thus the equilibrium transition function (12) is concave. This is the case

⁸ It is straightforward to show, from (12a) and (12b), that locus (b) lies above locus (a) for all $k_t < \tilde{k}$ and vice versa for all $k_t > \tilde{k}$.

drawn in Figure 2. However, if $\alpha > 0.5$ then the locus (a) is convex. In this case, the transition function is convex for all $k_t \in [0, \tilde{k})$, and concave for all $k_t \geq \tilde{k}$. This is the case drawn in Figure 3.

Steady State Equilibria

A steady state equilibrium is a competitive equilibrium in which $k_{t+1} = k_t = k$ in the transition function (12). Proposition 1 summarizes key properties of these equilibria.

Proposition 1: Steady state equilibria have the following properties.

- (i) If $\alpha < .5$ then there exist two steady state equilibria, one of which is degenerate and the other which has positive income and longevity. The degenerate steady state is unstable and the non-degenerate one is stable. The stable (non-degenerate) equilibrium may have $0 < p < 1$ or $p = 1$, depending on parameter values.
- (ii) If $\alpha > .5$ then the number of steady state equilibria depends on the value of A . The critical value \bar{A} is given by:

$$\bar{A} = \frac{e^{\frac{x(1-\alpha)}{\beta}} (1+\beta)^{\frac{(1+\beta)(1-\alpha)}{\beta}}}{\beta^\alpha \alpha^{1-\alpha} (1-\alpha)^\alpha}$$

If $A < \bar{A}$ then the unique steady state equilibrium is degenerate. If $A > \bar{A}$ then three steady state equilibria exist, two of which are stable. Of the stable equilibria, one is degenerate and the other has positive income and longevity with $p = 1$.

Proof: See Appendix A.

If $\alpha < 0.5$, then only one stable steady state equilibrium exists, and it has a strictly positive value of k . There are two cases. In Figure 2, this steady state is represented by the point B , where the locus (b) intersects the 45 degree line at k_b . This figure is drawn under the assumption that the critical value \tilde{k} is smaller than k_b . Hence, in this steady state equilibrium, all agents choose not to exit: $p = 1$, as is implicitly assumed in Diamond's model. However, as illustrated in Figure 4, the model also allows for the possibility that the locus (a) intersects the 45 degree line (at point A) before it intersects locus (b). In this case, the steady state equilibrium is at point A , with capital stock $k_a < \tilde{k}$. In this equilibrium, in every period, some fraction of agents chooses to exit: $p < 1$. In both cases, however, the unique stable steady state equilibrium has a positive income level – the degenerate steady state equilibrium at $k = 0$ is unstable.

If, however, $\alpha > 1/2$, then multiple stable steady state equilibria may exist. The locus (a) is convex, so the transition function in (12) is convex for $k < \tilde{k}$, and is concave for $k \geq \tilde{k}$, beyond the point where locus (a) intersects locus (b). Figure 3 illustrates how this can generate multiple steady states, when $A > \bar{A}$.⁹ In this case, three steady state values of k exist: 0, k_a , and k_b , where $0 < k_a \leq k_b$. The intermediate steady state, k_a , is unstable and the other two are stable. Starting with any $k_0 > k_a$ yields a transition path to k_b ; whereas starting with $k_0 < k_a$ yields a transition path to the origin. The steady state at the origin is stable and is the worst outcome with extreme poverty and minimum life expectancy of $1 + p_t = 1$. This poverty trap is a *death trap* for poor economies. The poor exit after the period of youth and do not save, perpetuating extreme poverty.¹⁰ Finally, when $A < \bar{A}$, the (a) locus is so low that it crosses the 45

⁹ Notice that this condition can be re-written as a restriction on x , given A , rather than the way we have expressed it here (and in Proposition 1). Written in this alternative way, multiple steady states exist if x is below a threshold value (identified in the formula in part (ii) of the Proposition). We chose to express this condition in terms of A because it is more straightforward to do.

¹⁰ The model gives the stark result that in the poverty trap capital and wages are zero. In this dire situation, young agents, if they had the choice, might prefer to exit at the beginning of the period of youth. To avoid this possibility, one could extend the model by allowing young agents access to a

degree line beyond the point k_b in Figure 3. In this case, the unique steady state equilibrium is at the origin – and, as before, this equilibrium is stable.

The basic model also has interesting implications for the path of wages and interest rates even ignoring the possibility of poverty traps. Notice, for example, that, when $\alpha < .5$, the transition path in Figures 2 and 4 initially lies on locus (a), which is below the transition path (b) for Diamond’s model. Thus starting from a small k_0 , the basic model displays higher interest rates, lower aggregate savings, and hence lower rates of wage growth on the transition path – as long as locus (a) lies below locus (b). Even if the transition path eventually joins the Diamond transition path at $p = 1$, the overall time to converge to the steady state will be longer than in Diamond’s model.

Income and Longevity

We now consider the implications that this model has for the relationship between per capita income and average longevity. In the model agents all live through youth and the proportion p_t live through old age. Thus, the average longevity of agents born at t (i.e., the average *cohort life expectancy*) in the model is $1 + p_t$. From Lemma 2 we know that there is a threshold level of capital, \tilde{k} , above which $p_t = 1$ and below which $p_t = p_t^* < 1$. Defining y_t as income per worker, we can then identify $\tilde{y} = A(\tilde{k})^\alpha$ as the threshold value of y corresponding to \tilde{k} . Using Lemma 2, we have the following new lemma.

sufficiently attractive primitive technology that needs only labour (e.g. hunting/gathering or simple agriculture). The poverty trap equilibrium in this extended model involves society de-industrializing so that there is a switch to the more primitive technology.

Lemma 3. For any given configuration of parameters (α, β, A, x) , the equilibrium relationship between income per worker y_t and average longevity of agents $1+p_t$ born in period t , is:

$$0 \leq y_t < \tilde{y} \Rightarrow 1+p_t = 1+p_t^* = 1 + \left(\frac{\rho_2}{\rho_1} \right) ((1-\alpha)y_t)^{\frac{\alpha}{1-\alpha}} < 2$$

$$y_t \geq \tilde{y} \Rightarrow 1+p_t = 2.$$

For $y_t < \tilde{y}$, average longevity is increasing in y_t and is strictly concave if $\alpha < 0.5$, strictly convex if $\alpha > 0.5$, and linear if $\alpha = 0.5$.

Figure 5 illustrates the case when $\alpha < 0.5$; here, longevity is increasing and strictly concave in income per worker below the threshold $\tilde{y} = A(\tilde{k})^\alpha$, and constant, at the maximum longevity of 2 periods, for all $y_t > \tilde{y}$.

The concave relationship in Figure 5 resembles the ‘‘Preston curve’’: the empirical relationship named after Preston (1975), and studied somewhat extensively. However, the comparison is only suggestive because, empirically, the Preston curve is usually expressed with different variables on the axes. The vertical axis measures life expectancy, but does so using current survivorship data. That is, in the data, life expectancy in period t is represented by $1+p_{t-1}$. Also, the horizontal axis typically measures income *per capita*, rather than income *per worker*. Moreover, income per capita averages output over the young and the old in period t : $y_t/(1+p_{t-1})$.

Lemma 4. For any given configuration of parameters (α, β, A, x) , the equilibrium relationship at time t between the average per capita income, $y_t/(1+p_{t-1})$, and life expectancy, $1+p_{t-1}$, is increasing and strictly convex until the lifespan of $1+p_{t-1}=2$ is reached.

Proof: See Appendix A.

Along the transition path in this model, as in the Preston curve, life expectancy and per capita income move together until a threshold income level is reached – beyond which life expectancy is constant. However, the model predicts a strictly convex relationship in these measured variables whereas the Preston curve is usually described as concave. Recall, though, that our basic model assumes 100% depreciation. In Appendix B, we show that, with incomplete depreciation, the Preston curve starts initially convex but may become concave as income increases.¹¹ This is broadly consistent with, for example, Deaton’s (2003) estimation of the Preston curve, using population-weighted nonparametric regression techniques.

Comparative Dynamics: The Effects of Changing x

The parameter x affects the equilibria in intuitive ways. First consider the case (i), with $\alpha < 0.5$, in Proposition 1, where there is a unique stable steady state. Figure 6 illustrates the effect of an increase in the utility value to exiting from x to x' . Suppose, initially, with x , the economy is in a steady state at point B on locus (b) and therefore $p = 1$ (the same as in Figure 2). An increase in x shifts the (a) locus downwards, but will leave the (b) locus unaffected. This will lead to an increase in \tilde{k} . If the increase is large enough, as in Figure 6, with the increase to x' , the economy will move to a qualitatively different equilibrium, similar to that in Figure 4, where a fraction of agents chooses to exit after the change. As agents exit this lowers the capital stock, which reduces the wage and induces more exit until a lower steady state capital is achieved at point A', where $p_t < 1$ and $k_a < \tilde{k}'$.

Alternatively, in case (ii), when $\alpha > 0.5$, raising x sufficiently high can result in $A < \bar{A}$ so that the only steady state equilibrium is the degenerate one. This could

¹¹ The results of Lemma 3 are also sensitive to the size of the depreciation rate. When $\alpha < 0.5$, we find the income-longevity relationship is initially concave as in Figure 5 but, depending on parameter values, can become convex.

cause a catastrophic decline in an economy which, for example, was originally in a steady state with positive capital.

Viewing this somewhat differently, if an economy is currently in an equilibrium such as point A in Figure 4, where some fraction of the population chooses not to live out their whole lives, then a change in peoples' beliefs – reducing the value of x – could lead to growth in the medium run and increase per capita income and life expectancy in the long run.¹²

Thus, it is possible in this model to generate a positive relationship between per capita income and life expectancy, in steady states, by considering different values of x . In principle, this could be thought of as another interpretation of the Preston curve. However, this equilibrium relationship between per capita income and life expectancy clearly reaches a critical point when $p = 1$ and no further reductions in x will have any impact on either life expectancy or per capita income in the steady state. Moreover, it seems unrealistic to think of more advanced economies as being those with more pessimism about the payoff from exit.¹³

¹² Thus, for example, St. Augustine's decision to make suicide a sin, and Thomas Aquinas' decision to re-emphasize this, (making it illegal) may have had stimulative effects on the Christian economies of the times.

¹³ Alternatively, one could consider the Preston curve to be generated by different values of A in the steady state equilibrium of this model. This interpretation is more plausible, perhaps, and has the added benefit that, beyond a threshold value of A , further increments increase per capita output without affecting life expectancy (which is at its maximum of 2). Once again, though, this generates a convex Preston curve, up to the point of the maximum p . Details are available from the authors upon request.

4. EFFICIENT ALLOCATIONS

We now turn to consider efficient allocations in this model. Here, we allow agents to make the existential choice of exiting – we consider only allocations where the planner is restricted by individuals' exit decisions.

From the utility function we can determine how consumption allocations affect individual exit choices and, thus, population choices:

- if $x > \beta u(c_{2t})$, then $I_{t-1} = 1$ and $p_{t-1} = 0$;
- if $x = \beta u(c_{2t})$, then $0 \leq I_{t-1} \leq 1$ and $0 \leq p_{t-1} \leq 1$;
- if $x < \beta u(c_{2t})$, then $I_{t-1} = 0$ and $p_{t-1} = 1$.

The aggregate feasibility condition at time t is:

$$F(K_t, L_t) + (1 - \delta)K_t \geq K_{t+1} + N_t c_{1t} + p_{t-1} N_{t-1} c_{2t-1}.$$

With 100% depreciation and a constant population, we have following intensive relationship

$$f(k_t) - k_{t+1} \geq c_{1t} + p_{t-1} c_{2t-1}.$$

As a benchmark for the analysis, we consider the golden-rule allocation. With non-satiation, the feasibility condition holds with equality, and in the steady state production efficiency requires $f'(k^*) = 1$. With production efficiency, the feasibility condition becomes:

$$c_1 + p c_2 = f(k^*) - k^*.$$

To determine exchange efficiency we compare the planner's problem over the three ranges of c_2 corresponding to the above exit decisions. First, when $x < \beta u(c_2)$, no

agent exits, $I = 0$ and $p_t = 1$. This is the standard problem, as in Diamond (1965), and optimality requires:

$$u'(c_1) = \beta u'(f(k^*) - k^* - c_1).$$

When $x > \beta u(c_2)$, all agents exit $I = 1$ and $p_t = 0$. The planner's problem is:

$$\max_{c_1} u(c_1) + x \quad \text{s.t.} \quad f(k^*) - k^* = c_1$$

so that $c_1 = f(k^*) - k^*$.

In the last case, $x = \beta u(c_2)$, agents are indifferent about exiting. The planner's problem reduces to:

$$\max_{c_1} u(c_1) + x \quad \text{s.t.} \quad f(k^*) - k^* = c_1 + pu^{-1}(x/\beta)$$

so that $c_1 = f(k^*) - k^* - pu^{-1}(x/\beta)$.

This last allocation is clearly dominated by the second one, where all agents exit. It is inefficient for the planner to choose a positive c_2 such that $x = \beta u(c_2)$, because agents who do not exit receive no more utility than those that do exit but they use up resources. It is more efficient to lower c_2 and thereby encourage all agents to exit. The freed resources are used to increase consumption in youth and thereby overall utility.

Thus, the effective choice the planner faces is between the Diamond case or the 100% exit case. Whichever case is optimal depends on the value of x . There is clearly a threshold value of x , denoted x^* , beyond which all agents exit. The following proposition summarizes.

Proposition 2. The golden-rule comprises both production and exchange efficiency conditions. Production efficiency requires a k value such that $f'(k^*) = 1$. Exchange efficiency depends on the exit value x relative to a threshold value x^* . For $x \leq x^*$, the planner sets $c_1^* = f(k^*) - k^*$ and $c_2^* = 0$. In this case, all agents exit: $p = 0$. For $x > x^*$, the planner sets $u'(c_1^*) = \beta u'(c_2^*)$ and $c_1^* + c_2^* = f(k^*) - k^*$. In this case, no agents exit: $p = 1$.

The golden rule requires $p = 0$ or $p = 1$. But, as demonstrated in Section 3, steady state equilibria may, under certain circumstances, have $p < 1$. This implies that these equilibria are inefficient.

Corollary. All steady-state equilibria with $p < 1$ are inefficient.

The source of this inefficiency is different from that in the standard overlapping generations model with production, and is unrelated to capital over-accumulation. Consider, for example, a steady state allocation where $k = k^*$ but $p \in (0,1)$. By the above reasoning this is inefficient even though production efficiency is achieved. Thus, the inefficiency isn't coming from the fact that there is too much capital investment but, rather, due to the fact that there is not enough first period consumption when agents are indifferent about exiting in the competitive steady state. Thus, the inefficiency is due to the number of individuals that choose not to exit.

5. EXTENDING THE MODEL

This section generalizes the model to consider public health care (as in Chakraborty (2004)) and to allow agents to choose savings and exit probabilistically. We relate the very similar choices of exiting and not investing in health and show two main results. The first is that individuals would not choose interior probabilities of exit (i.e., the discrete choice analysed above is not restrictive). The second result is that allowing for investment in health can reinforce the exit mechanism and generate multiple equilibria with a lower threshold value of α than in the base model.

As in Chakraborty (2004) we define a biological survival probability $\phi \in [0,1]$ which is realized at the beginning of the second period of life, and where $\phi(h_t)$ is increasing and concave in public health care expenditures h_t : $\phi'(h_t) > 0$, $\phi''(h_t) < 0$, $\phi(0) > 0$.

This probability enters consumer expected utility as follows:

$$U_t = \begin{cases} u(c_{1t}) + [\phi(h_t)\beta u(c_{2t+1}) + (1 - \phi(h_t)) \cdot d] & \text{if } I_t = 0 \\ u(c_{1t}) + x & \text{if } I_t = 1 \end{cases}$$

where d is the present value associated with exiting life by illness. In general, we can think of the value of d as being distinct from x . For example, $d < x$ could represent a situation where death from poor health comes through a painful illness that the agent would prefer not to experience. Alternatively, values of $d > x$ might describe situations where wilful exit is seen as sinful, so that death through natural causes is preferred to death by one's own hand. In any event, for wilful exit to be an optimal choice, d must be small enough so that $x > \phi(h_t)\beta u(c_{2t+1}) + (1 - \phi(h_t))d$.

Notice that, with $0 < \phi(h_t) < 1$, we have a contingency that does not arise in the basic model. An agent may optimally choose to *not* exit, and so choose positive savings, but

then die from illness anyway at the end of period 1. We assume, for simplicity, that these agents' savings are discarded.¹⁴

Probabilistic Choices and Equilibrium

Here, we model agents as choosing the probability of exiting. From the perspective of an individual, h_t (and, hence, $\phi(h_t)$) is given exogenously. Let e_t denote the probability of an agent exiting ($I_t = 1$) and $1 - e_t$ is the probability of not exiting ($I_t = 0$). Then expected utility can be written:

$$U_t = u(c_{1,t}) + e_t \cdot x + (1 - e_t)[\phi(h_t)\beta u(c_{2,t+1}) + (1 - \phi(h_t)) \cdot d]$$

Recall, exit is at the end of the period of youth after the savings decision. If an agent exits there are no further decisions. Otherwise, if the agent doesn't exit, the agent trivially spends all his savings in his second period of life: $c_{2,t+1} = R_{t+1}s_t$, in the usual way. Knowing this, agents born in period $t = 0, 1, 2, \dots$ choose savings, (s_t) , $(c_{1,t})$, and e_t , to maximize utility U_t subject to $c_{1,t} + s_t = w_t$ and $c_{2,t+1} = R_{t+1}s_t$. The following proposition characterizes the solution of this optimization problem.

Proposition 3. Given h_t , w_t and R_{t+1} , it is individually optimal to either choose to:

- (i) exit and not save, $e_t = 1$ and $s_t = 0$, or
- (ii) not exit and save, $e_t = 0$ and $s_t = \frac{\beta w_t}{(1 + \beta)}$.

Choosing $e_t \in (0, 1)$ is strictly inferior.

Proof. See Appendix A.

¹⁴ However, as before, we get the same general results if we use an annuity market as in Chakraborty (2004).

Choosing any probability of $e_t \in (0,1)$ is dominated by certainty, due to the joint decision about savings.^{15 16} This proposition is proven assuming that the exit and savings decisions are made simultaneously. However, it also goes through if the exit decision is either before or after the savings decision. If the probabilistic exit decision is before the end of the period, the analysis is implicitly assuming a mechanism for precommitment. This might correspond to lifestyle choices made early in the first period.¹⁷

Public Health Expenditure

We now examine what happens in equilibrium when h_t is determined, at the aggregate level, by the government's budget constraint. As in Chakraborty (2004), health expenditure here is financed by a proportional wage tax so that $h_t = \tau w_t$, where $\tau > 0$ is the wage tax rate. The following specific function builds on Chakraborty's example.

$$\phi(h_t) = \begin{cases} \phi^o + \frac{\sigma h_t}{1+h_t}, & \forall 0 \leq h_t < \bar{h} \\ 1, & \forall h_t \geq \bar{h} \end{cases}$$

¹⁵ When we allow that $h_t = 0$ yields certain death $\phi_t(0) = 0$, the results change slightly. Given $h_t = 0$, the optimal choice requires $s_t = 0$ and (i) $e_t = 1$ if $x > d$, (ii) $e_t = 0$ if $x < d$, or (iii) $e_t \in [0,1]$. Still, in all cases, restricting e_t to be discrete is nonbinding on optimizing behaviour. This is equivalent to choosing $I_t = 0$ or 1 non-probabilistically.

¹⁶ This proposition is robust to small errors in picking exit. If an agent chose $e_t = 1$ but knew that there was a small probability that exit wouldn't happen, then they would save a small amount as insurance against the ghastly prospect of zero consumption. Still, they would choose the corner solution.

¹⁷ Lifestyle choices could be simply modelled by make e_t an argument in the period utility function. For example, modeling first period utility $u(c_{1t}, e_t)$, where $u_e > 0$ captures the possibility that exit is associated with a "carefree" lifestyle that gives greater first period utility *ceteris paribus*.

where $\bar{h} = \frac{1-\phi^o}{\sigma-(1-\phi^o)}$, $0 < \phi^o < 1$ and $\sigma > (1-\phi^o)$. The constant ϕ^o is the probability of living when $h_t = 0$; i.e. when health care expenditures are zero. The function $\phi(h_t)$ is strictly concave in up to the maximum value $\phi_t = 1$ corresponding to $h_t = \bar{h}$.

Lemma 2 can now be readily generalized, using an analogous proof.

Lemma 5: For any given configuration of parameters (α, β, A, x, d) , and given value of h , a threshold value of capital stock \tilde{k}_h exists, and the equilibrium relationship between the proportion of savers p_t (i.e. those that do not exit) and capital per worker k_t satisfies:

$$p_t = p_t^* \Leftrightarrow 0 \leq k_t < \tilde{k}_h \quad (13a)$$

$$p_t = 1 \Leftrightarrow k_t \geq \tilde{k}_h, \quad (13b)$$

where $p_t^* = \left(\frac{\rho_{2h}}{\rho_{1h}} \right) \left[(1-\alpha)(1-\tau)Ak_t^\alpha \right]^{\alpha/1-\alpha}$ is the internal proportion of savers, and the coefficients ρ_{1h} and ρ_{2h} are defined by:

$$\rho_{1h} \equiv \frac{\beta\phi(h)}{1+\beta\phi(h)} > 0 \quad \text{and} \quad \rho_{2h} \equiv \frac{(\beta\phi(h))^{\frac{1}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}}}{e^{\frac{x-(1-\phi(h))d}{\beta\phi(h)(1-\alpha)}} (1+\beta\phi(h))^{\beta\phi(h)(1-\alpha)}} > 0.$$

Including health expenditure in the model changes its solution in three ways. First, the effective discount rate changes from β to $\beta\phi$; second, disposable income is now $w_t - h = w_t(1-\tau)$; third, the difference between the two exit utilities, $x - (1-\phi)d$, affects the equilibrium proportion of savers. Because h and ϕ are exogenous to the individual, the results in Lemmas 3 and 4 obtain here; i.e. the longevity is strictly concave in wage income if $\alpha < 1/2$, and the Preston curve is always strictly convex.

From Lemma 5 the transition function can be written as:

$$k_{t+1} = \begin{cases} \rho_{2h} ((1-\alpha)Ak_t^\alpha)^{\frac{1}{1-\alpha}}, & \forall 0 \leq k_t < \tilde{k}_h \\ \rho_{1h} (1-\alpha)(1-\tau)Ak_t^\alpha, & \forall k_t \geq \tilde{k}_h \end{cases} \quad (14a)$$

$$(14b)$$

Simulations reveal that (14a) can be strictly convex when $\alpha < 0.5$ (e.g. $\alpha = 0.43$, $\beta = 0.3$, $\tau = 0.1$, $A = 80$, $x = d = 1$). Therefore, starting from a low level of the capital stock the economy moves up along a convex locus (14a), and when capital stock grows sufficiently, the transition function switches to the concave locus (14b). This convex-concave transition curve may yield multiple equilibria as illustrated in Figure 6. The fact that the threshold value of α for multiple equilibria is now reduced indicates that public health and the individual's choice to exit can interact in a way to reinforce each other.

6. CONCLUSIONS

In this paper we have extended the standard (Diamond (1965)) overlapping generations model with production to allow agents the choice of whether or not to live out the second period of their lives. Agents' perceptions of the value of exiting life early are crucial in this decision problem. In this paper we modelled this simply as a parameter that is common for all agents: x . We found that it is relatively straightforward to introduce this parameter into the model, and the results from doing so are (we think) quite interesting. First, we have shown that, for any value of $x \in \mathbb{R}$, and for any value of α (capital's share of income), given logarithmic utility, there always exists a range of (low) capital stock values where some agents would choose *not* to live out their whole lives. Thus, Diamond's original model, where agents are presumed to live out their entire lives, is restrictive – at least for a certain class of utility functions that includes the logarithmic function.

Secondly, once this choice is allowed, the shape of the transition function and the number of steady state equilibria now depend crucially on the value of α : capital's share of income. If, as is typically presumed, $\alpha < 0.5$, then the transition function is concave and the economy has a unique stable steady state equilibrium with a strictly positive value of income per capita. This case is similar to Diamond's but, if the initial capital stock is below a critical value \tilde{k} , some agents will choose to exit along the transition path to the steady state. Moreover, in the steady state equilibrium, some agents may still choose to exit, depending on beliefs about the value of doing so: x .

When $\alpha > 0.5$ this model has a convex equilibrium transition function for low values of k . This feature of the model makes its predictions quite different from those of the standard Diamond model: it implies that the degenerate steady state equilibrium is stable, and it introduces the possibility of multiple steady states. If x is small enough, then another stable steady state exists, with positive k and where all agents choose to live out their entire lives. Otherwise, the *only* stable steady state equilibrium is the degenerate one – where nobody chooses to live out their second period.¹⁸

Efficiency conditions in this model are similar to those in Diamond's, with a comparable golden rule, but with an extra condition: either no-one should live out their second period or everyone should. This condition comes about because, in any equilibrium where some fraction of agents chooses to exit, individuals are indifferent about whether or not they are alive in the second period. Since being alive in the second period is expensive (it costs some of the consumption good) a rational planner would not allow it if agents are indifferent anyway.

The model also generates equilibrium relationships between longevity and per capita income. In particular, as an economy develops, according to this model, in equilibrium it will experience higher levels of per capita income and higher *average*

¹⁸ The relevance of the results, for $\alpha > 0.5$, depend, of course, on one's view about the size of α empirically. Estimates of this parameter depend on how narrowly one defines capital.

values of longevity, until a maximum longevity is reached. If one is willing to interpret international income and life expectancy data as observations along this path, then the model generates a relationship that is comparable to the Preston curve. This relationship is important to understand when considering measures of development that include both variables.¹⁹

How important is the exit decision to economic growth and development? One application that seems particularly relevant is to indigenous communities in countries with European colonizers. For example, in Australia, the suicide rate for aboriginals is approximately double that of others in the country and the gap in life expectancy between these two groups is approximately 17 years, with aboriginal males having a life expectancy of only 59 years.²⁰ Lifestyle choice can play a major role in determining health status and life expectancy, and this is linked to overall economic well-being.²¹ These communities typically suffer from chronically low rates of investment and growth.

Similarly, falling incomes in former Soviet bloc countries, over the period 1989-1999 have been linked with significantly higher suicide rates, alcoholism, death rates from accidents, and a drop in life expectancy for Russian men from 62 (in 1980) to 58 (in 1999).²² Finally, in sub-Saharan Africa, many countries have been experiencing long periods of negative growth – and these are precisely the countries with low life expectancy.

¹⁹ See, for example, Usher (1980) and Becker *et al* (2005).

²⁰ Department of Health and Ageing (Australia) (2006) *Factbook* 2006.

²¹ Pincus *et al* (1998) argue that self management and social conditions (eg., education, job, and lifestyles) are more important to health and adult longevity than access to health care.

²² Suicide rates increased by 60%, 80%, and 95% in Russia, Lithuania, and Latvia, respectively, over the period 1989-1999, and the death rate from accidents (many of them involving alcohol) increased in Russia by 83% from 1991-1999, according to the United Nations Development Report *Transition 1999*.

We believe that this model sheds some light on the issue of longevity and growth, but is too simple to be offered as a quantitative theory of this issue. Other features would need to be added in order to take the model more seriously empirically. For example, it is well-known that life expectancy figures are influenced significantly by infant mortality numbers. To take this into account, this model would need to be extended by introducing a third period of life: childhood, as in (for example) Blackburn and Cipriani (2002). Secondly, as Finlay (2006) points out, individuals also face trade-offs between health expenditures and human capital investments, which can also be important. Finally, different individuals surely have different perceptions of the value of x , and this could potentially play a significant role in their decisions. We consider each of these avenues as interesting, and leave all of them to future research.

APPENDIX A

Proof of Lemma 2.

We first establish that, in any time t , there exists at least one positive value of k_t , call it \tilde{k} , for which the equality in (5) holds with $p_t = 1$. Substitution of the firms' first order conditions (6) and (7) into the right hand side of (5) yields the following expression in k_t and k_{t+1} :

$$\beta \ln((1 - \alpha)Ak_t^\alpha) + \beta \ln(\alpha Ak_{t+1}^{\alpha-1}) + \beta \ln \beta - (1 + \beta) \ln(1 + \beta)$$

Setting $p_t = 1$ in (10) and substituting the result into the above expression yields the following function $g(k_t)$:

$$g(k_t) = \beta \ln((1 - \alpha)Ak_t^\alpha) + \beta \ln(\alpha A((1 - \alpha)\frac{\beta}{1 + \beta})Ak_t^\alpha)^{\alpha-1}) + \beta \ln \beta - (1 + \beta) \ln(1 + \beta)$$

Differentiating:

$$g'(k_t) = \frac{\beta \alpha^2}{k_t} > 0$$

Thus, since the right hand side of (5) is strictly increasing in k_t and the left hand side is constant, there exists a unique value of k_t (\tilde{k}) beyond which further increments in k_t lead to strict inequality in (5). Hence, if $k_t > \tilde{k}$ then $p_t = 1$, as in (11b). Similarly, if $p_t = 1$, then, by Lemma 1, the right hand side of (5) must be no less than the left hand side and, since $g'(k_t) > 0$, by the definition of \tilde{k} , it must be that $k_t \geq \tilde{k}$. Hence, if $p_t = 1$ then $k_t \geq \tilde{k}$, as in (11b). The value of \tilde{k} , given in the Lemma, is found by solving the equation $g(\tilde{k}) = x$.

Now consider an equilibrium $k_t \in (0, \tilde{k})$. We first show that p_t cannot equal either 1 or 0 – it must be interior. We then show that an interior p_t implies (11a). Suppose $p_t = 1$, i.e. all young agents choose to stay in the model, since $g(k_t)$ is an increasing function of k_t , with $k_t < \tilde{k}$ and the definition of \tilde{k} , the left-hand side of (5) is always bigger than the right-hand side, so the optimal strategy is to exit not to stay (a contradiction). Similarly, suppose $p_t = 0$, i.e., all young agents choose to exit the model, from (10) this case leads to a zero intensive capital k_{t+1} in period $t+1$. From the firm's first order condition (7), this implies (in the limit) unbounded R_{t+1} and thus the inequality in (5) is strict for any $x \in \mathbb{R}$. Therefore agents would rather stay than exit the model in this case (a contradiction). Thus, for any given $k_t < \tilde{k}$, in equilibrium, $p_t \in (0,1)$. If $p_t \in (0,1)$, then agents must be indifferent about staying or exiting, hence, (5) must hold with equality. Substitution of (10) into (5), with equality, yields (11a).

Finally, if $k_t = 0$, then, from (6), $w_t = 0$, and no income is earned in period t . Thus, no capital is available for period $t+1$. In this case, due to the term $\beta \ln w_t$, the right hand side of (5) is unbounded from below, and condition (5) is violated for any $x \in \mathbb{R}$. The optimal strategy is to exit: $p_t = 0$. Similarly, if $p_t = 0$ then it must be that $k_t = 0$. Suppose not, so that $p_t = 0$ and $k_t > 0$. Any deviator who saves would receive unbounded returns, so that (5) would hold as a strict inequality, for any $x \in \mathbb{R}$. Hence, the deviator would not exit and $p_t > 0$ (a contradiction). This implies (11c). ■

Proof of Proposition 1

Consider the following functions, from the right hand side of equations (12a) and (12b) respectively:

$$l_a(k_t) = \rho_2 \left((1-\alpha) A \right)^{\frac{1}{1-\alpha}} k_t^{\frac{\alpha}{1-\alpha}}$$

$$l_b(k_t) = \rho_1 (1-\alpha) A k_t^\alpha$$

It is straightforward to show that $l_b(k_t)$ lies above $l_a(k_t)$ for all $k_t < \tilde{k}$ and vice versa for all $k_t > \tilde{k}$. Also, $l_a(k_t) = l_b(k_t)$ at two values of k_t : 0 and \tilde{k} . The following properties of $l_b(k_t)$ are well known: $l_b(0) = 0$, $l'_b(k_t) > 0$, $l''_b(k_t) < 0$, $\lim_{k_t \rightarrow 0} l'_b(k_t) = \infty$, and $\lim_{k_t \rightarrow \infty} l'_b(k_t) = 0$. It is also straightforward to show that $l_a(k_t)$ has the following properties: $l_a(0) = 0$, $l'_a(k_t) > 0$. Moreover, if $\alpha < 0.5$ then $l''_a(k_t) < 0$, $\lim_{k_t \rightarrow 0} l'_a(k_t) = \infty$, and $\lim_{k_t \rightarrow \infty} l'_a(k_t) = 0$. Also, if $\alpha > 0.5$ then $l''_a(k_t) > 0$, $\lim_{k_t \rightarrow 0} l'_a(k_t) = 0$, and $\lim_{k_t \rightarrow \infty} l'_a(k_t) = \infty$.

To prove part (i) of the Proposition, we first consider the degenerate equilibrium. By (12a), we know that the relevant transition function for all $k_t \in [0, \tilde{k}]$ is given by $k_{t+1} = l_a(k_t)$. Since $l_a(0) = 0$, then $k = 0$ is a steady state equilibrium. To prove that this is unstable, consider now a deviation from this equilibrium $k_t = \varepsilon > 0$ and $\varepsilon < \tilde{k}$, then k_{t+1} is given by $l_a(k_t)$. Since, when $\alpha < 0.5$, $\lim_{k_t \rightarrow 0} l'_a(k_t) = \infty$ then there exists a small $\varepsilon > 0$ such that $l'_a(\varepsilon) > 1$. Thus, using (12a), $k_{t+1} > k_t = \varepsilon$. Hence, the degenerate equilibrium is unstable.

We now consider two cases of non-degenerate steady state equilibria: when $p = 1$, and when $p = p^* < 1$. If $p = 1$ then, by (11b) and (12b), the steady state equilibrium value k_b is given by $k_b = l_b(k_b)$. Solving this equation, we obtain:

$$k_b = (\rho_1(1-\alpha)A)^{\frac{1}{1-\alpha}}$$

It is easy to show that there exists an A such that $k_b > \tilde{k}$. Hence, if k_b is a steady state equilibrium then $p = 1$. To prove that this equilibrium is stable, note that $\lim_{k_t \rightarrow 0} l_b'(k_t) = \infty$, and that $l_b(k_t)$ is continuous and strictly concave. Hence, $l_b'(k_b) < 1$. That is, $l_b(k_t)$ crosses the 45° line from above. Hence, $k_t \in (0, k_b) \Rightarrow k_{t+1} > k_t$ and $k_t > k_b \Rightarrow k_{t+1} < k_t$. Hence, the steady state equilibrium at k_b is stable.

If $p = p^* < 1$ then, by (11a) and (12a), the steady state equilibrium value k_a is given by $k_a = l_a(k_a)$. Solving this equation, we obtain:

$$k_a = \left(\rho_2 \left((1-\alpha)A \right)^{\frac{1}{1-\alpha}} \right)^{\frac{1-\alpha}{1-2\alpha}}$$

It is easy to show that there exists an A such that $k_a < \tilde{k}$. Hence, if k_a is a steady state equilibrium then $p = p^* < 1$. To prove that this equilibrium is stable, note that, when $\alpha < 0.5$, $\lim_{k_t \rightarrow 0} l_a'(k_t) = \infty$, and that $l_a(k_t)$ is continuous and strictly concave. Hence, $l_a'(k_a) < 1$. That is, $l_a(k_t)$ crosses the 45° line from above. Hence, $k_t \in (0, k_a) \Rightarrow k_{t+1} > k_t$ and $k_t > k_a \Rightarrow k_{t+1} < k_t$. Hence, the steady state equilibrium at k_a is stable.

To prove part (ii) of the Proposition, we first consider the degenerate equilibrium. By (12a), we know that the relevant transition function for all $k_t \in [0, \tilde{k})$ is given by $k_{t+1} = l_a(k_t)$. Since $l_a(0) = 0$, then $k = 0$ is a steady state equilibrium. To prove that

this is stable, consider now a deviation from this equilibrium $k_t = \varepsilon > 0$ and $\varepsilon < \tilde{k}$, then k_{t+1} is given by $l_a(k_t)$. Since, when $\alpha > 0.5$, $\lim_{k_t \rightarrow 0} l'_a(k_t) = 0$ then there exists a small $\varepsilon > 0$ such that $l'_a(\varepsilon) < 1$. Thus, using (12a), $k_{t+1} < k_t$, and the degenerate equilibrium is stable.

We now consider two cases of non-degenerate steady state equilibria: when $p = 1$, and when $p = p^* < 1$. If $p = 1$ then, by (11b) and (12b), the steady state equilibrium value k_b is given by $k_b = l_b(k_b)$. This is entirely equivalent to the steady state equilibrium, analysed above, in case (i) when $\alpha < 0.5$, and $p = 1$. Hence, there exists an A such that $k_b = (\rho_1(1-\alpha)A)^{\frac{1}{1-\alpha}}$ is a stable steady state equilibrium with $p = 1$.

If $p = p^* < 1$ then, by (11a) and (12a), the steady state equilibrium value k_a is given by $k_a = l_a(k_a)$. Solving this equation, as before, we obtain:

$$k_a = \left(\rho_2 \left((1-\alpha)A \right)^{\frac{1}{1-\alpha}} \right)^{\frac{1-\alpha}{1-2\alpha}}$$

The following condition identifies values of A for which $k_a < \tilde{k}$:

$$k_a < \tilde{k} \Leftrightarrow A > \bar{A} = \frac{e^{\frac{x(1-\alpha)}{\beta}} (1+\beta)^{\frac{(1+\beta)(1-\alpha)}{\beta}}}{\beta^\alpha \alpha^{1-\alpha} (1-\alpha)^\alpha}$$

Hence, if k_a is a steady state equilibrium then $p = p^* < 1$. To prove that this equilibrium is unstable, note that, when $\alpha > 0.5$, $\lim_{k_t \rightarrow 0} l'_a(k_t) = 0$, and that $l_a(k_t)$ is continuous and strictly convex. Hence, $l'_a(k_a) > 1$. That is, $l_a(k_t)$ crosses the 45° line from below. Hence, $k_t \in (0, k_a) \Rightarrow k_{t+1} < k_t$ and $k_t > k_a \Rightarrow k_{t+1} > k_t$. Hence, the steady state equilibrium at k_a is unstable. Finally, notice that, if $A < \bar{A}$, so that $k_a > \tilde{k}$, then

(by (12a) and (12b)) k_a is not a steady state equilibrium. Hence, in this case, the only steady state equilibrium is the degenerate one. ■

Proof of Lemma 4

Output per worker is $y_t = Ak_t^\alpha$. The relationship between k_t and k_{t-1} is given by (10).

Using this relationship and $p_{t-1}^* = \left(\frac{\rho_2}{\rho_1}\right) \left[(1-\alpha)Ak_{t-1}^\alpha\right]^{\frac{\alpha}{1-\alpha}}$, from (11a), yields:

$$\begin{aligned} y_t &= A \left[p_{t-1}^* \frac{\beta}{1+\beta} (1-\alpha)Ak_{t-1}^\alpha \right]^\alpha = A \left[p_{t-1}^* \frac{\beta}{1+\beta} (p_{t-1}^*)^{\frac{1-\alpha}{\alpha}} \left(\frac{\rho_1}{\rho_2}\right)^{\frac{1-\alpha}{\alpha}} \right]^\alpha \\ &= A \left(\frac{\beta}{1+\beta}\right)^\alpha \left(\frac{\rho_1}{\rho_2}\right)^{1-\alpha} p_{t-1}^* = A\rho_1(\rho_2)^{\alpha-1} p_{t-1}^* = \mu p_{t-1}^*, \end{aligned}$$

where $\mu = A\rho_1(\rho_2)^{\alpha-1} > 0$.

Now, define $z_t \equiv \frac{y_t}{1+p_{t-1}^*}$ as per capita income. Thus, we have:

$$z_t(p_{t-1}^*) = \frac{\mu p_{t-1}^*}{1+p_{t-1}^*} = \mu p_{t-1}^* (1+p_{t-1}^*)^{-1}$$

Simplifying notation:

$$z(p) = \mu p (1+p)^{-1}$$

Totally differentiating:

$$dz = [\mu(1+p)^{-1} - \mu p(1+p)^{-2}] dp$$

Hence:

$$\frac{dp}{dz} = \frac{1}{\mu(1+p)^{-1} - \mu p(1+p)^{-2}} > 0$$

Hence:

$$\begin{aligned}
\frac{d^2 p}{dz^2} &= \frac{d\left(\frac{dp}{dz}\right)}{dp} \cdot \frac{dp}{dz} \\
&= -\left(\mu(1+p)^{-1} - \mu p(1+p)^{-2}\right)^2 \left(-\mu(1+p)^{-2} - \mu(1+p)^{-2} + 2\mu p(1+p)^{-3}\right) \left(\mu(1+p)^{-1} - \mu p(1+p)^{-2}\right)^{-1} \\
&= -\left(\mu(1+p)^{-1} - \mu p(1+p)^{-2}\right)^3 \left(-\mu(1+p)^{-2} - \mu(1+p)^{-2} + 2\mu p(1+p)^{-3}\right) > 0
\end{aligned}$$

■

Proof of Proposition 3

The Kuhn-Tucker conditions to the household problem corresponding to the maximization problem yield three nontrivial possible solutions, two corner solutions and one interior solution. As before we use $u(c) = \ln(c)$.

Case 0: $e_t = 0$ and $s_t = \frac{\beta\phi(h_t)(w_t - h_t)}{1 + \beta\phi(h_t)}$.

Case 1: $e_t = 1$ and $s_t = 0$. If $s_t = 0$, then $c_{2t+1} = 0$ and $U \rightarrow -\infty$ unless $e_t = 1$. Conversely, if $I_t = 1$, then $s_t = 0$, maximizes $U = \ln(w_t - h_t - s_t) + x$.

Case 2: $e_t = 1 - \frac{e^{\frac{x-(1-\phi)d}{\beta\phi}}}{\beta\phi(h_t)[(w_t - h_t)R_{t+1} - e^{\frac{x-(1-\phi)d}{\beta\phi}}]}$ and $s_t = \frac{e^{\frac{x-(1-\phi)d}{\beta\phi}}}{R_{t+1}}$.

This interior solution is feasible when:

$$0 < \frac{e^{\frac{x-(1-\phi)d}{\beta\phi}}}{\beta\phi(h_t)[(w_t - h_t)R_{t+1} - e^{\frac{x-(1-\phi)d}{\beta\phi}}]} < 1 \quad \text{and} \quad 0 < \frac{e^{\frac{x-(1-\phi)d}{\beta\phi}}}{R_{t+1}} < w_t - h_t. \quad (15)$$

To find the optimum we compare the utility levels:

$$U_{0t} = \ln\left[\frac{w_t - h_t}{1 + \beta\phi(h_t)}\right] + \beta\phi(h_t) \ln\left[\frac{\beta\phi(h_t)(w_t - h_t)R_{t+1}}{1 + \beta\phi(h_t)}\right];$$

$$U_{1t} = \ln(w_t - h_t) + x;$$

$$U_{2t} = \ln\left[(w_t - h_t) - \frac{e^{\frac{x-(1-\phi)d}{\beta\phi}}}{R_{t+1}}\right], \quad \text{if (15) is satisfied.}$$

Clearly, when Case 2 is feasible $U_{1t} > U_{2t}$. The interior solution never optimal, and the agent's optimal decision on her longevity reduces down to $e_t = 0$ or $e_t = 1$. ■

APPENDIX B

The body of the paper assumes 100% depreciation. This appendix develops the analysis for a general depreciation rate $\delta \in [0,1]$. Depreciation does not affect our generic results when $\alpha > .5$. With $\alpha < .5$, we were unable to prove Proposition 1 (i) analytically. However, extensive numerical simulations uncovered no exceptions. Lowering δ can change the previous strictly concave relationship between wage and longevity when $\alpha < .5$ described in Lemma 3 to one that is first concave and then convex.

The depreciation rate affects the user cost of capital and enters the first-order condition for capital in the firm's problem yielding real interest rate $R_{t+1} = 1 - \delta + \alpha A k_{t+1}^{\alpha-1}$. The equilibrium condition is unchanged $k_{t+1} = p_t s_t$, and after substituting for savings (2) and the wage rate (6), the gross interest rate is:

$$R_{t+1} = 1 - \delta + \alpha A \left(p_t \frac{\beta}{1 + \beta} (1 - \alpha) A k_t^\alpha \right)^{\alpha-1}.$$

Recall that (5) describes the condition where agents prefer not to exit. Setting $p_t = 1$ and substituting R_{t+1} into the inequality (5) and taking the antilog yields:

$$(1-\alpha)(1-\delta)Ak_t^\alpha + (1-\alpha)^\alpha \alpha \left(\frac{\beta}{1+\beta} \right)^{\alpha-1} A^{\alpha+1} k_t^{\alpha^2} \geq \frac{e^{\frac{x}{\beta}} (1+\beta)^{\frac{1+\beta}{\beta}}}{\beta}.$$

The left-hand side of this inequality is strictly increasing in k_t . Since the right-hand side is a positive number, there exists a unique threshold value \tilde{k} where the expression holds with equality. The above inequality is therefore satisfied for any $k_t \geq \tilde{k}$. In another words, when $k_t > \tilde{k}$, $p_t = 1$ and the transition function is given by

$$k_{t+1} = \frac{\beta}{1+\beta} (1-\alpha) Ak_t^\alpha.$$

On the other hand if $0 < k_t < \tilde{k}$, the equilibrium will settle down at $p_t \in (0,1)$ and (5) holds with strict equality. Substituting the gross interest rate into (5) for $p_t < 1$ yields:

$$(1-\alpha)(1-\delta)Ak_t^\alpha + (1-\alpha)^\alpha \alpha \left(\frac{\beta}{1+\beta} \right)^{\alpha-1} A^{\alpha+1} k_t^{\alpha^2} p_t^{\alpha-1} = \frac{e^{\frac{x}{\beta}} (1+\beta)^{\frac{1+\beta}{\beta}}}{\beta}.$$

This condition gives us the proportion of savers:

$$p_t = \left(\frac{\alpha A \left(\frac{\beta}{1+\beta} \right)^{\alpha-1} \left((1-\alpha) Ak_t^\alpha \right)^\alpha}{\frac{e^{\frac{x}{\beta}} (1+\beta)^{\frac{1+\beta}{\beta}}}{\beta} - (1-\delta)(1-\alpha) Ak_t^\alpha} \right)^{\frac{1}{1-\alpha}}.$$

The equilibrium transition path of the economy with a general depreciation rate now can be summarized as the following:

$$k_{t+1} = \begin{cases} \left(\frac{\alpha A (1-\alpha) A k_t^\alpha}{\frac{e^{\frac{x}{\beta}} (1+\beta)^{\frac{1+\beta}{\beta}}}{\beta} - (1-\delta)(1-\alpha) A k_t^\alpha} \right)^{\frac{1}{1-\alpha}}, & \forall k_t < \tilde{k} & \text{(B1a)} \\ \rho_1 (1-\alpha) A k_t^\alpha, & \forall k_t \geq \tilde{k} & \text{(B1b)} \end{cases}$$

With $\alpha > 0.5$, the transition function is first described by the convex locus (B1a) and then switches to a concave locus (B1b). For a sufficiently large value of A , we get the situation depicted in Figure 3. Thus, we get the same generic results as in the body of the paper.

With $\alpha < 0.5$, the curvature of (B1a) can be ambiguous when $\delta < 1$ whereas it is strictly concave when $\delta = 1$. This implies that the economy may have multiple equilibria in contrast to the unique equilibrium when $\delta = 1$. We were unable to prove uniqueness in for $\delta < 1$. However, extensive numerical simulations failed to generate multiple equilibria. Thus, we conjecture that there is a unique equilibrium, in which case, Proposition 1 generalizes to $\delta \in [0, 1]$.

The depreciation rate can affect the empirical implications. For a general depreciation rate, the equilibrium relationship between the wage w_t and average longevity of agents $1 + p_t$ born at t , is:

$$0 \leq w_t < \tilde{w} \Rightarrow 1 + p_t = 1 + p_t^* = 1 + \left(\frac{\alpha A \left(\frac{\beta}{1+\beta} \right)^{\alpha-1} w_t^\alpha}{\frac{e^{\frac{x}{\beta}} (1+\beta)^{\frac{1+\beta}{\beta}}}{\beta} - (1-\delta) w_t} \right)^{\frac{1}{1-\alpha}} < 2$$

$$w_t \geq \tilde{w} \Rightarrow 1 + p_t = 2.$$

Below the threshold wage, \tilde{w} , the average longevity is increasing in w_t and is strictly convex for $\alpha > 0.5$. For $\alpha < 0.5$, the relationship is strictly concave when $\delta = 1$, the result in Lemma 3. However, when $\delta < 1$, the relationship may be concave and then convex (for example: $\delta = 0.1$, $\alpha = 0.3$, $\beta = 1$, $x = 0.6$, and $A = 1$).

The equilibrium relationship at time t between the average per capita income, $y_t/(1+p_{t-1})$, and life expectancy, $1+p_{t-1}$, is increasing until the lifespan of $1+p_{t-1}=2$ is reached for all depreciation rates. As described in Lemma 4, this relationship is strictly convex when $\delta = 1$. When $\delta < 1$ this relationship's shape is initially strictly convex but then its shape appears ambiguous. With a systematic search across the parameter space, we have been unable to find an example where the curve becomes concave, consistent with the usual description of the Preston curve.

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FIGURES

Figure 1:

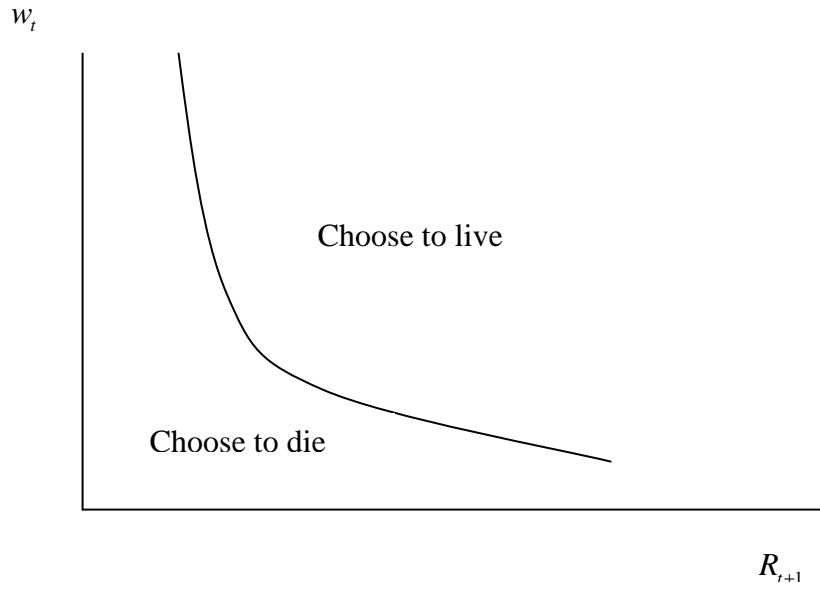
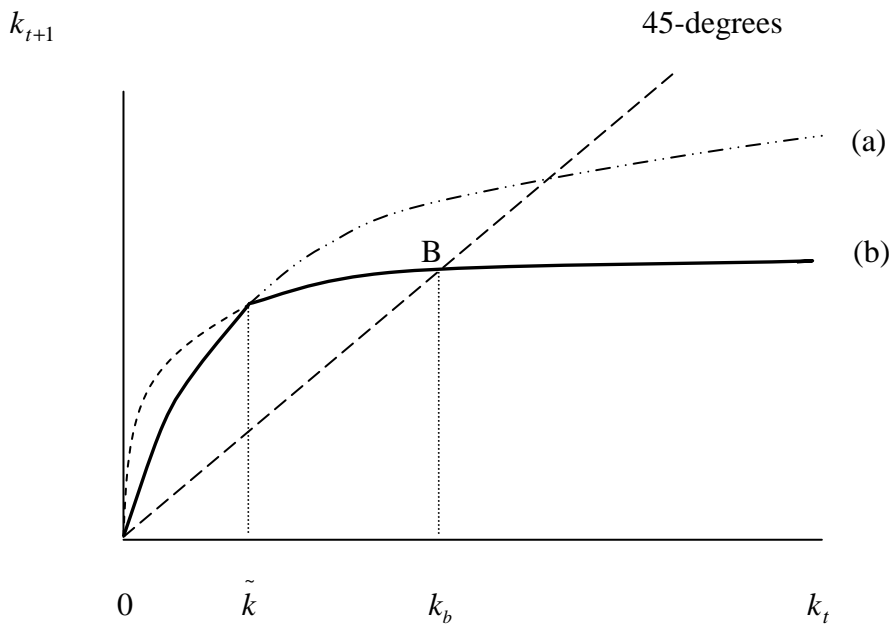


Figure 2:²³



²³ The solid black lines in Figures 2-4 and 6 represent the equilibrium transition functions.

Figure 3:

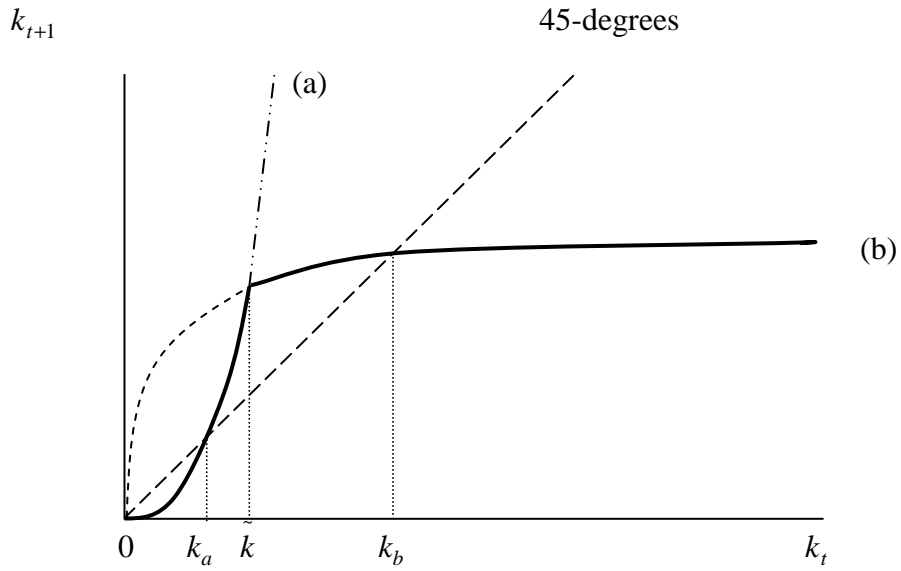


Figure 4

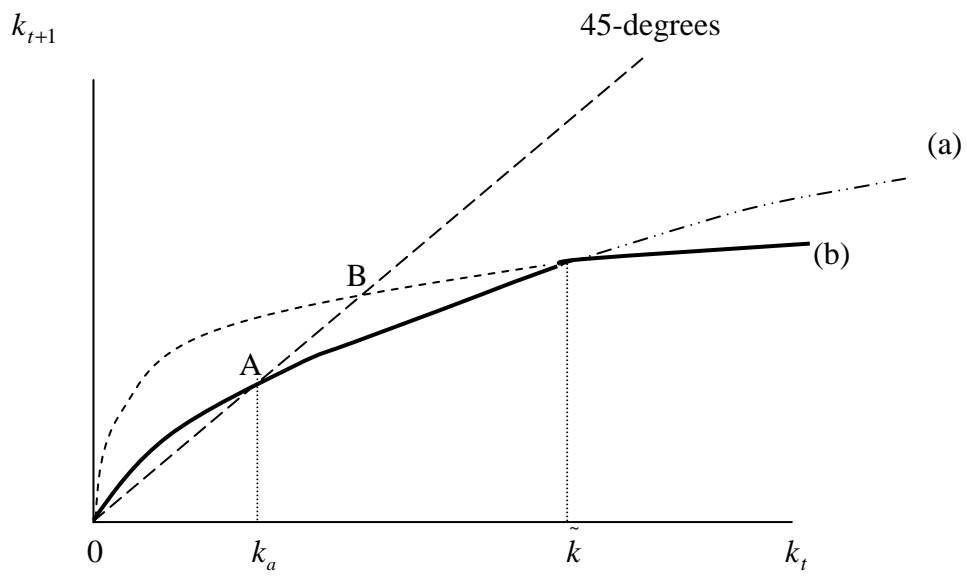


Figure 5:

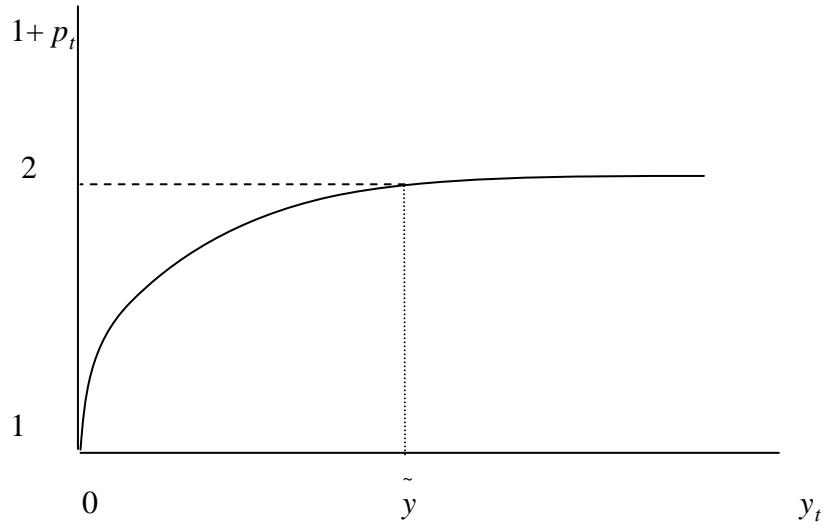
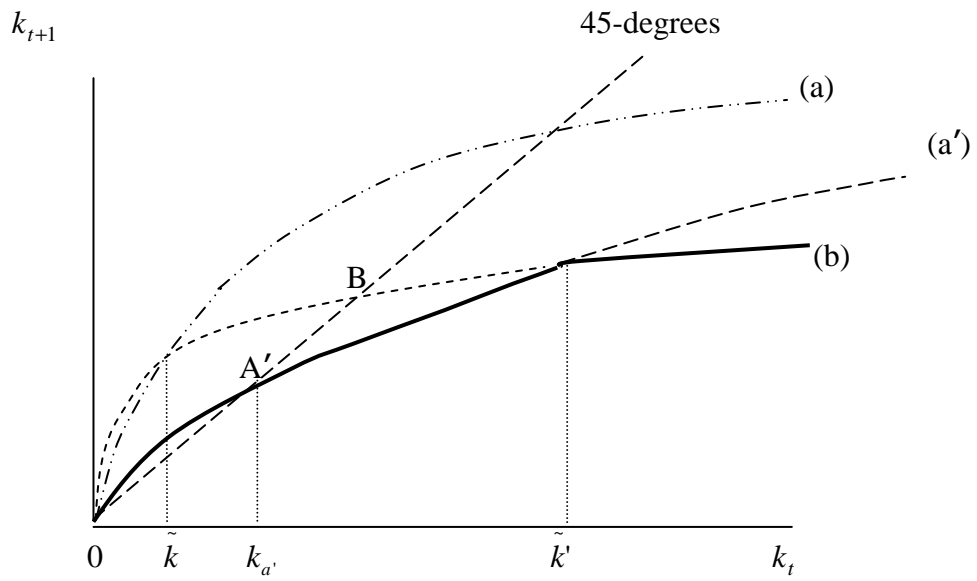


Figure 6²⁴



²⁴ The curve shifts from (a) to (a') with an increase in the exit utility $x' > x$.